



## Maths Policy

Policy date: Sept 25

## Intention for Mathematics across school

At Ingol Community Primary School our intention is to enable each pupil to develop, secure and deepen their level of mathematical understanding with a view to securing the mathematics skills and understanding required for later life, as well as creating an enthusiasm and fascination about maths itself. To help us achieve this, we have adopted Red Rose Maths and use the mastery approach to learning.

## The National Curriculum for Mathematics

*The 2014 national curriculum for Mathematics* aims to ensure that all pupils:

- become fluent in the fundamentals of Mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their Mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The 2014 National Curriculum programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The expectation is that **the majority of pupils** will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

## Early Years Foundation stage

*Following the 2021 reform to the EYFS framework*, developing a strong grounding in number is essential so that all children develop the necessary building blocks to excel mathematically. Children should be able to count confidently, develop a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers. By providing frequent and varied opportunities to build and apply this understanding – such as using manipulatives, including small pebbles and tens frames for organising counting – children will develop a secure base of knowledge and vocabulary from which mastery of mathematics is built. In addition, it is important that the curriculum includes rich opportunities for children to develop their spatial reasoning skills across all areas of mathematics including Shape, Space and Measures. It is important that children develop positive attitudes and interests in mathematics, look for patterns and relationships, spot connections, 'have a go', talk to adults and peers about what they notice and not be afraid to make mistakes.

## Cross-curricular

Mathematics teaches children how to make sense of the world around them through developing their ability to calculate, reason and solve problems. It is a core subject with a range of cross-curricular links but most often, is best taught discretely, using opportunities from other subjects to rehearse skills in a context. Numeracy involves developing confidence and competence in number work; shape, space and measure; handling data and the using and applying of these skills.

## Computing

Computing can enhance the teaching of Mathematics significantly. It has ways of impacting on learning that are not possible with conventional methods. Teachers can use software to present information visually, dynamically and interactively, so that children understand concepts more quickly. A range of software and resources are available to support work with the computers.

## Assessment and recording

Assessment for Learning is fundamental to raising standards and enabling children to reach their potential. Assessment in Mathematics takes place daily using a range of strategies such as marking and feedback of work and verbal discussions with children. This information informs subsequent planning and next steps in teaching and learning. Planning documentation is annotated to demonstrate adaptations and provide feedback about children's individual/group progress.

The Mathematics Subject Co-ordinator meets termly with selected children from each class, who are of differing mathematical ability, to discuss their thoughts on maths and review the work they have completed. Teachers meet termly to review individual samples of work against national expectations and moderate judgements. This demonstrates work at various levels of achievement in Mathematics from across the school to help support teacher's in making their own judgements with regards to attainment and progress.

Targets are set at the beginning of each year and progress towards them are regularly reviewed throughout the year. Records are collated to inform the school's School Improvement Plan (SIP) and Maths Action Plan. This tracking also includes half-termly tracking of standards for each child. This data is used by the Maths Subject Co-ordinator and Headteacher to review performance against national expectations for pupil age and progress towards end of year targets.

The outcomes of regular assessments tests are recorded and forwarded to the Maths Subject Co-ordinator. Formal assessments specific to year groups:

Year	Assessment
Foundation stage	Reception Baseline (Statutory)
Years 1 and 2	Teacher assessment
Years 3, 4, 5	Teacher assessment
Year 6	KS2 SATs (statutory)

## Reporting

Parent consultation evenings are held in the Autumn and Spring terms where children's progress and achievement will be discussed. All parents receive an end of year report in the Summer term on which there is a summary of their child's achievements and progress, together with a comment on the child's effort and engagement with mathematics. Short reports on progress are also provided for parents at the end of each full term.

The National Curriculum places great emphasis upon developing a greater breadth of skills and knowledge, so that pupils are confident to apply their skills to different areas of the curriculum. As such, when we report to parents, we will refer to the child's progress and current level of attainment in comparison to 'age related expectations' (the expected range of attainment for his/her age), using phrases such as:

- **Below** (working below age related expectations)
- **Just below** (just below or beginning to work at age related expectation)
- **Expected - On track** (working at age related expectation)
- **Deeper level** (working above age related expectation)

Our aim is for the majority of pupils to be working within age related expectations by the end of the academic year.

## Resources

All classrooms have a number of maths resources which are used to support pupils in their mathematical learning. Topic specific resources (such as weights and scales) are located in well-labelled central storage areas. There is a **whole school Calculation Policy (APPENDIX 1)**.

## Equalities

We believe that equality at our school should permeate all aspects of school life and is the responsibility of every member of the school and wider community. We will always strive to ensure equality of access to maths for all pupils irrespective of their gender, ethnicity, disability, religious beliefs/faith tradition, sexual orientation, age or any other of the protected characteristics (Single Equalities Act 2010)

## Inclusion

Wherever possible we aim to fully include all pupils in maths teaching. Through our maths teaching we provide learning opportunities that enable **all** pupils to make progress. We set suitable learning challenges and respond to each child's individual needs.

We will identify which pupils or groups of pupils are under-achieving and take steps to improve their attainment. More able children will be identified and suitable learning challenges provided. In lessons a variety of approaches will be used to ensure continuity and progression. Lessons will include a blend of whole class teaching, group work and individual work.

## Pupil Premium

All staff have knowledge of Pupil Premium children in their class. Support staff are utilised to ensure that these pupils have the support they need to make good progress – this may be in a group of pupils with SEND, in a group of low achievers or in a group of very able pupils.

## Roles and Responsibilities

### The School Leadership Team

- To actively support and encourage staff, praising good practise and supporting staff development, in-service training and resources.
- To monitor teaching and learning through lesson observations, learning walks and work scrutiny, and to give informative and constructive feedback.
- Support staff development through training and provision of resources.

### Subject Co-ordinator

- To work with the Headteacher and the Senior Leadership Team to monitor, plan and develop the subject to allow for progression, continuity and high standards of attainment in Mathematics.
- To support colleagues in the teaching of Mathematics and provide a strategic lead and direction in the subject.
- To manage periodic book reviews to ensure the curriculum is being covered and the marking policy is adhered to.
- To undertake brief termly Learning Walks with an agreed focus *ie Working Walls, Times Tables*
- To gather Pupil Voice information from agreed children and particular groups *ie SEND, PP etc*
- To monitor progress in Mathematics, highlight and plan actions required.
- To take responsibility for auditing and organising Mathematics resources.
- To keep up to date with developments in Mathematics education and to inform colleagues as appropriate.
- To draw up annual action plan for Mathematics.
- To review the school policy for Mathematics as appropriate.

### The Class Teacher

- To be responsible for the planning and teaching of Mathematics
- To manage and supervise their class use of Mathematics equipment.

### **The Governors**

- The named governor has responsibility to oversee the progress of Mathematics in school. They will meet with the subject leader a minimum of once per term to review development plans.

Signed: *J Moss* (Maths Subject leader)

September 2025

Review: September 2026

# APPENDIX 1

## Guidance paper – Calculation

### Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, and written methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave primary school they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

### Mental methods of calculation

Oral and mental work in Mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall and use key number facts instantly – for example, all addition and subtraction facts for each number to at least 20 (Year 2) and multiplication facts up to  $12 \times 12$  (Year 4);
- use taught strategies to work out the calculation – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number or a multiple of 10 to a one-digit or two-digit number (Year 1) and partition two-digit numbers in different ways including into multiples of ten and one and add the tens and ones separately and then recombine (Year 2);
- understand how the rules and laws of arithmetic are used and applied – for example, to add or subtract mentally combinations of one-digit and two-digit numbers (Year 2), and to calculate mentally with whole numbers and decimals (Year 6).

## Written methods of calculation

The 2014 National Curriculum progression in written methods of calculation highlights how children should move from informal methods of recording to expanded methods that are staging posts to a compact written method for each of the four operations.

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. The curriculum promotes the use of what are commonly known as ‘standard’ written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

The incidence of children moving between schools and localities is very high in some parts of the country. Moving to a school where the written method of calculation is unfamiliar and does not relate to that used in the previous school can slow the progress a child makes in Mathematics. There will be differences in practices and approaches which can be beneficial to children. However, if the long-term aim is shared across all schools and if expectations are consistent then children’s progress will be enhanced rather than limited. The entitlement to be taught how to use efficient written methods of calculation is set out clearly in the renewed objectives. Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

## Objectives

The programmes of study in the new National Curriculum shows the progression in children’s use of written methods of calculation in the strands Number and place value, Addition and subtraction and Multiplication and division.

	Number and place value	Addition and subtraction	Multiplication and division
Year 1	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number</li> <li>count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens</li> <li>given a number, identify one more and one less</li> <li>identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least</li> <li>read and write numbers from 1 to 20 in numerals and words.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs</li> <li>represent and use number bonds and related subtraction facts within 20</li> <li>add and subtract one-digit and two-digit numbers to 20, including zero</li> <li>solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as <math>7 = - 9</math>.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</li> </ul>
Year 2	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward</li> <li>recognise the place value of each digit in a two-digit number (tens, ones)</li> <li>identify, represent and estimate numbers using different representations, including the number line</li> <li>compare and order numbers from 0 up to 100; use <math>&lt;</math>, <math>&gt;</math> and <math>=</math> signs</li> <li>read and write numbers to at least 100 in numerals and in words</li> <li>use place value and number facts to solve problems.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>solve problems with addition and subtraction: <ul style="list-style-type: none"> <li>using concrete objects and pictorial representations, including those involving numbers, quantities and measures</li> <li>applying their increasing knowledge of mental and written methods</li> </ul> </li> <li>recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</li> <li>add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> <li>a two-digit number and ones</li> <li>a two-digit number and tens</li> <li>two two-digit numbers</li> <li>adding three one-digit numbers</li> </ul> </li> <li>show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot</li> <li>recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers</li> <li>calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (<math>\times</math>), division (<math>\div</math>) and equals (=) signs</li> <li>show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot</li> <li>solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.</li> </ul>

	Number and place value	Addition and subtraction	Multiplication and division
Year 3	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number</li> <li>recognise the place value of each digit in a three-digit number (hundreds, tens, ones)</li> <li>compare and order numbers up to 1000</li> <li>identify, represent and estimate numbers using different representations</li> <li>read and write numbers up to 1000 in numerals and in words</li> <li>solve number problems and practical problems involving these ideas.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>add and subtract numbers mentally, including: <ul style="list-style-type: none"> <li>a three-digit number and ones</li> <li>a three-digit number and tens</li> <li>a three-digit number and hundreds</li> </ul> </li> <li>add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</li> <li>estimate the answer to a calculation and use inverse operations to check answers</li> <li>solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables</li> <li>write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods</li> <li>solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which <math>n</math> objects are connected to <math>m</math> objects.</li> </ul>
Year 4	<p>Pupils should be taught to</p> <ul style="list-style-type: none"> <li>count in multiples of 6, 7, 9, 25 and 1000</li> <li>find 1000 more or less than a given number</li> <li>count backwards through zero to include negative numbers</li> <li>recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)</li> <li>order and compare numbers beyond 1000</li> <li>identify, represent and estimate numbers using different representations</li> <li>round any number to the nearest 10, 100 or 1000</li> <li>solve number and practical problems that involve all of the above and with increasingly large positive numbers</li> <li>read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate</li> <li>estimate and use inverse operations to check answers to a calculation</li> <li>solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>recall multiplication and division facts for multiplication tables up to <math>12 \times 12</math></li> <li>use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers</li> <li>recognise and use factor pairs and commutativity in mental calculations</li> <li>multiply two-digit and three-digit numbers by a one-digit number using formal written layout</li> <li>solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as <math>n</math> objects are connected to <math>m</math> objects.</li> </ul>
	Number and place value	Addition and subtraction	Multiplication and division
Year 5	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit</li> <li>count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000</li> <li>interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers,</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)</li> <li>add and subtract numbers mentally with increasingly large numbers</li> <li>use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers</li> <li>Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers</li> <li>establish whether a number up to 100 is prime and recall prime numbers up to 19</li> <li>multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long</li> </ul>

	<p>including through zero</p> <ul style="list-style-type: none"> <li>• round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000</li> <li>• solve number problems and practical problems that involve all of the above</li> <li>• read Roman numerals to 1000 (M) and recognise years written in Roman numerals.</li> </ul>	<ul style="list-style-type: none"> <li>• solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.</li> </ul>	<p>multiplication for two-digit numbers</p> <ul style="list-style-type: none"> <li>• multiply and divide numbers mentally drawing upon known facts</li> <li>• divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</li> <li>• multiply and divide whole numbers and those involving decimals by 10, 100 and 1000</li> <li>• recognise and use square numbers and cube numbers, and the notation for squared (<sup>2</sup>) and cubed (<sup>3</sup>)</li> <li>• solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes</li> <li>• solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign</li> <li>• solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.</li> </ul>
Year 6	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>• read, write, order and compare numbers up to 10 000 000 and determine the value of each digit</li> <li>• round any whole number to a required degree of accuracy</li> <li>• use negative numbers in context, and calculate intervals across zero</li> <li>• solve number and practical problems that involve all of the above.</li> </ul>	<p>Pupils should be taught to:</p> <ul style="list-style-type: none"> <li>• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</li> <li>• divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</li> <li>• divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context</li> <li>• perform mental calculations, including with mixed operations and large numbers</li> <li>• identify common factors, common multiples and prime numbers</li> <li>• use their knowledge of the order of operations to carry out calculations involving the four operations</li> <li>• solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why</li> </ul>	

# Calculation Policy

## Progression Towards a Written Method for Addition

*At Ingol we follow the Red Rose Maths scheme created by Lancashire. The Lancashire Maths team (LEA) have produced a progression towards written methods of calculation so that they meet statutory requirements set out in the National Curriculum. At Ingol we have adopted this policy as a basis for progression when teaching written methods to our pupils from Reception up to Year Six*

*(Adopted from Lancashire Maths team)*

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e.  $5 + 3 = 3 + 5$
- associative i.e.  $5 + 3 + 7 = 5 + (3 + 7)$

The fact that it is commutative and associative means that calculations can be rearranged, e.g.  $4 + 13 = 17$  is the same as  $13 + 4 = 17$ .

### **RECEPTION**

#### **Early Learning Goal:**

***Using quantities and objects, children add two single-digit numbers and count on to find the answer.***

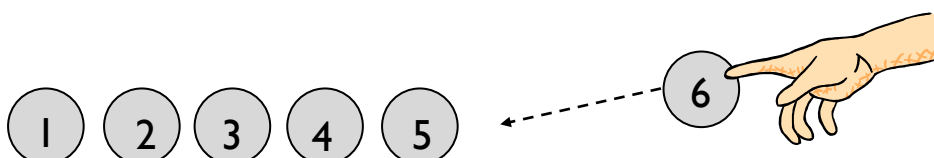
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

#### **Counting all method**

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating  $4 + 2$ , they are encouraged to count out four counters and count out two counters.



To find how many altogether, touch and drag them into a line one at a time whilst counting.



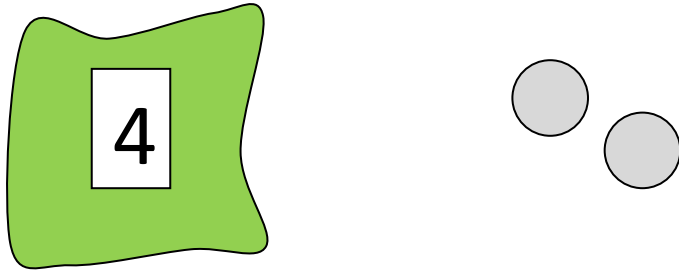
By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

**Counting on method**

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating  $4 + 2$ , count out the two groups of counters as before.



then cover up the larger group with a cloth.



For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

**Those who are ready** may record their own calculations.

**Y1**

**End of Year Objective:**  
*Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).*

Children will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on. Using their developing understanding of place value, they will move on to be able to use Base 10 equipment to make teens numbers using separate tens and units. For example, when adding 11 and 5, they can make the 11 using a ten rod and a unit.



The units can then be combined to aid with seeing the final total, e.g.



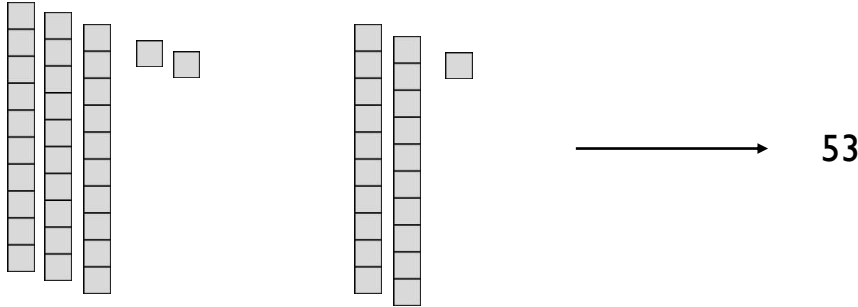
so  $11 + 5 = 16$ . If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.

## Y2

### End of Year Objective:

**Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers.**

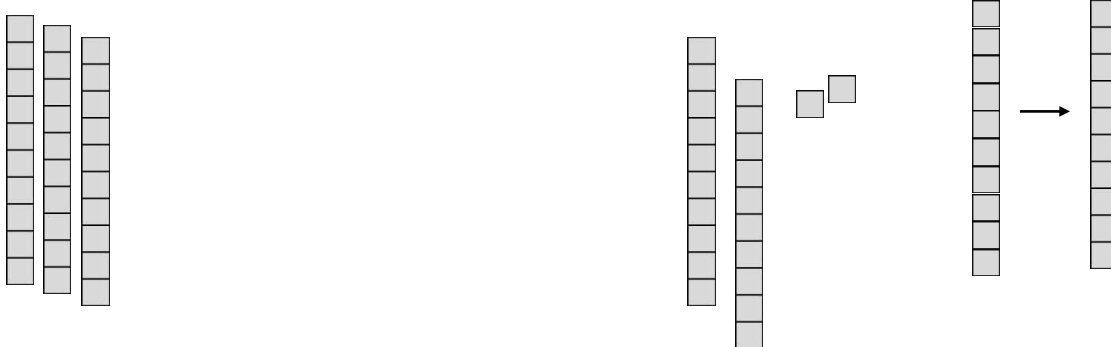
Children will continue to use the Base 10 equipment to support their calculations. For example, to calculate  $32 + 21$ , they can make the individual amounts, counting the tens first and then count on the units.



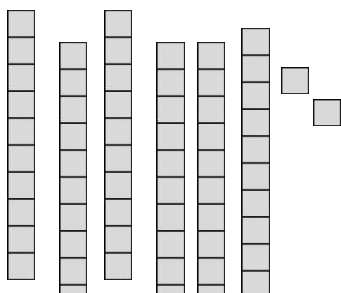
When the units total more than 10, children should be encouraged to exchange 10 units/ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition. For example, when calculating  $35 + 27$ , they can represent the amounts using Base 10 as shown:



Then, identifying the fact that there are enough units/ones to exchange for a ten, they can carry out this exchange:

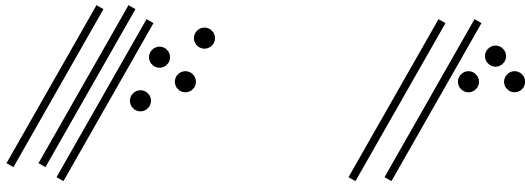


To leave:



Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks).

e.g.  $34 + 23 =$



With exchange:

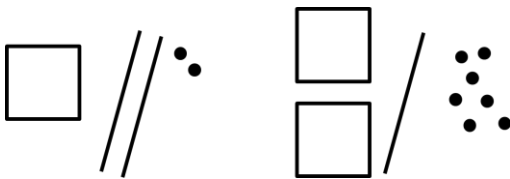
e.g.  $28 + 36 =$



so  $28 + 36 = 64$

It is important that children circle the remaining tens and units/ones after exchange to identify the amount remaining.

This method can also be used with adding three digit numbers, e.g.  $122 + 217$  using a square as the representation of 100.



### Y3

**End of Year Objective:**  
**Add numbers with up to three digits, using formal written method of columnar addition.\***

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

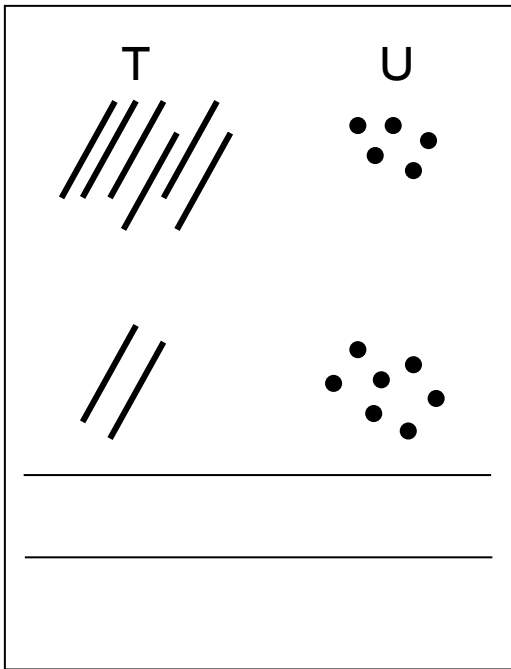
Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

Children should add the **least significant digits** first (i.e. start with the units/ones), and in an identical method to that from year 2, should identify whether there are greater than ten units which can be exchanged for one ten.

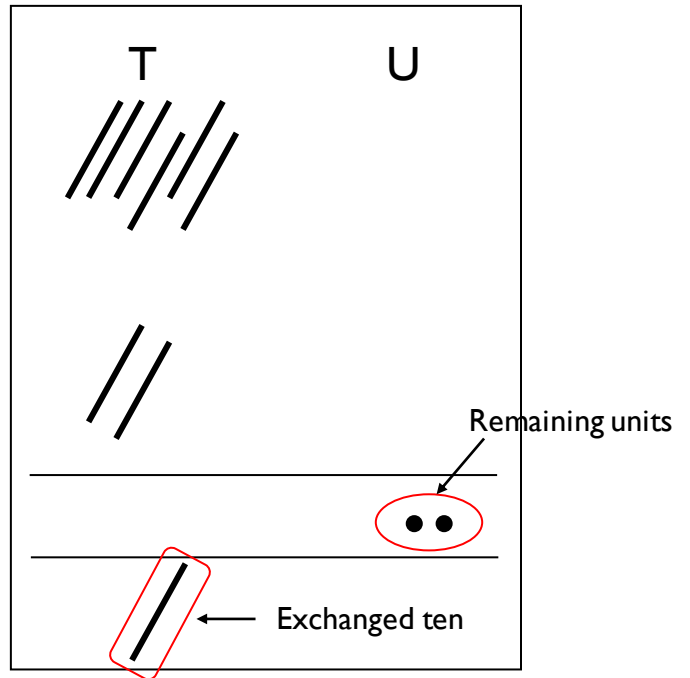
They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns (as in Step 1 in the diagram below).

e.g.  $65 + 27$

Step 1



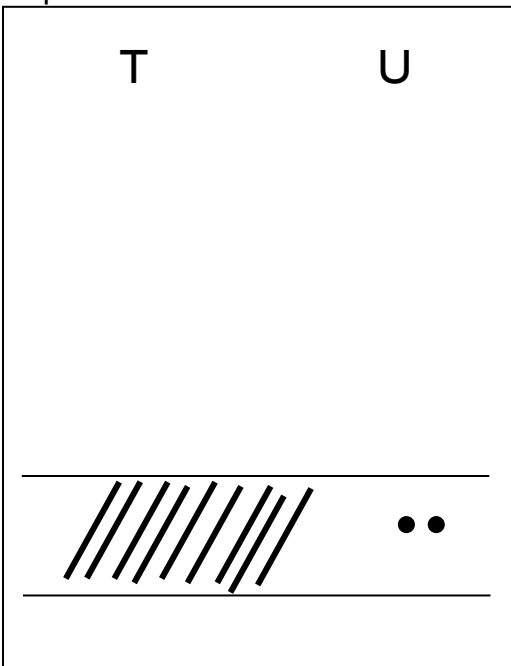
Step 2



Children would exchange ten units/ones for a ten, placing the exchanged ten below the equals sign. Any remaining units/ones that cannot be exchanged for a ten move into the equals sign as they are the units part of the answer (as in the diagram in Step 2 above).

If there are any tens that can be exchanged for a hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 below).

Step 3



Written method

Step 1	Step 2	Step 3																																						
<table border="0"> <tr><td>T</td><td>U</td></tr> <tr><td>6</td><td>5</td></tr> <tr><td>+ 2</td><td>7</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td colspan="2"><hr/></td></tr> </table>	T	U	6	5	+ 2	7	<hr/>		<hr/>		<table border="0"> <tr><td>T</td><td>U</td></tr> <tr><td>6</td><td>5</td></tr> <tr><td>+ 2</td><td>7</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>2</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>1</td></tr> </table>	T	U	6	5	+ 2	7	<hr/>			2	<hr/>			1	<table border="0"> <tr><td>T</td><td>U</td></tr> <tr><td>6</td><td>5</td></tr> <tr><td>+ 2</td><td>7</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td>9</td><td>2</td></tr> <tr><td colspan="2"><hr/></td></tr> <tr><td></td><td>1</td></tr> </table>	T	U	6	5	+ 2	7	<hr/>		9	2	<hr/>			1
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Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially.

This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method.

By the end of year 3, children should also extend this method for three digit numbers.

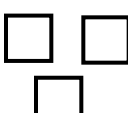




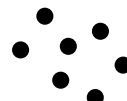
### Y4

**End of Year Objective:**

**Add numbers with up to 4 digits *and* decimals with one decimal place using the formal written method of columnar addition where appropriate.**

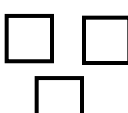

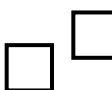



Children will move to year 4 using whichever method they were using as they transitioned from year 3.

#### Step 1

H	T	U
		
		

H	T	U
3	6	5
+	2	4

#### Step 2

H	T	U
		
		
		
		

H	T	U
3	6	5
+	2	4
		2
		1

Step 3

H	T	U
□ □		
□		
□ □		
□		• •

	H	T	U
	3	6	5
+	2	4	7
			2

Step 4

H	T	U
□ □ □ □ □ □		• •

	H	T	U
	3	6	5
+	2	4	7
	6		2

By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

## Y5

### End of Year Objective:

**Add whole numbers with more than 4 digits *and* decimals with two decimal places, including formal written methods (columnar addition).**

Children should continue to use the carrying method to solve calculations such as:

$$\begin{array}{r} 3364 \\ + 247 \\ \hline 3611 \\ \hline \end{array}$$

$$\begin{array}{r} 3121 \\ + 148 \\ \hline 3306 \\ \hline \end{array}$$

$$\begin{array}{r} 3.56 \\ + 2.47 \\ \hline 6.03 \\ \hline \end{array}$$

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

## Y6

### End of Year Objective:

**Add whole numbers and decimals using formal written methods (columnar addition).**

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

$$\begin{array}{r} 6432 \\ 786 \\ + 4681 \\ \hline 11944 \\ \hline \end{array}$$

$$\begin{array}{r} 401.20 \\ + 26.85 \\ + 0.71 \\ \hline 428.76 \\ \hline \end{array}$$

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

## Progression Towards a Written Method for Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of addition
- not commutative i.e.  $5 - 3$  is not the same as  $3 - 5$
- not associative i.e.  $10 - 3 - 2$  is not the same as  $10 - (3 - 2)$

### **YR**

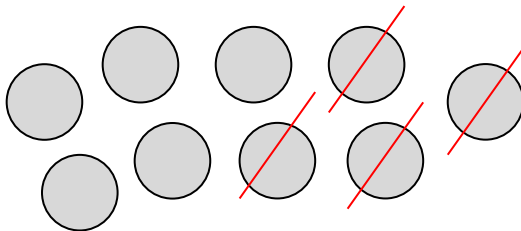
#### **Early Learning Goal:**

**Using quantities and objects, children subtract two single-digit numbers and count on or back to find the answer.**

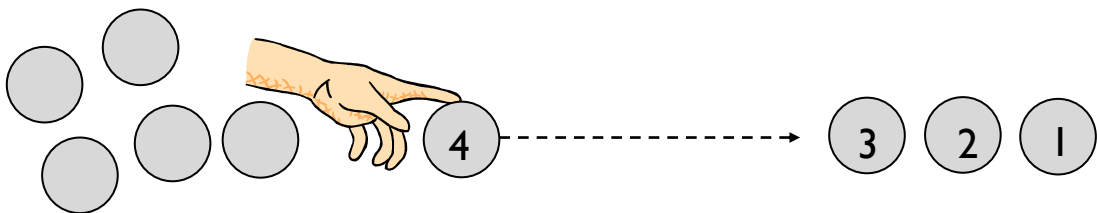
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

#### **Taking away**

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g.  $9 - 4$ .



For illustration purposes, the amount being taken away are show crossed out. Children would be encouraged to physically remove these using touch counting.



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

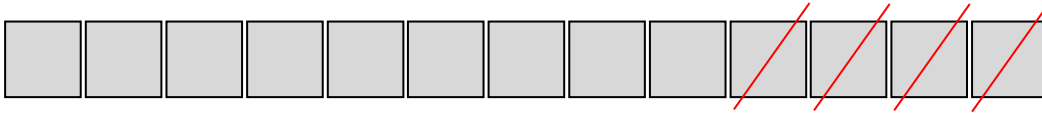
**Those who are ready** may record their own calculations.

## Y1

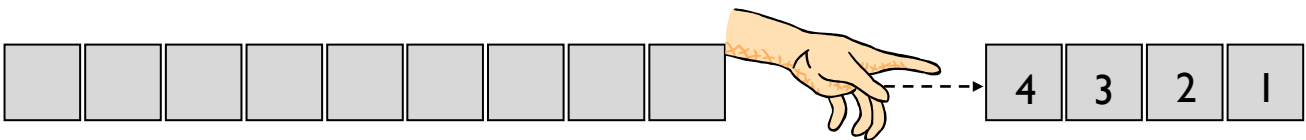
### End of Year Objective:

**Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).**

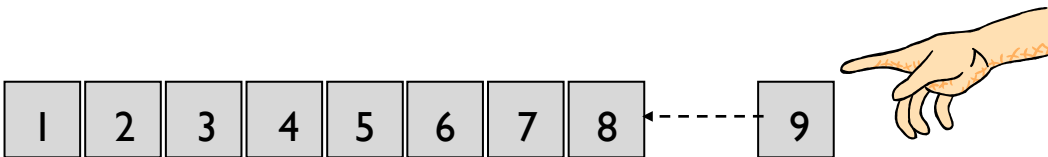
Children will continue to use practical equipment and taking away strategies. To avoid the need to exchange for subtraction at this stage, it is advisable to continue to use equipment such as counters, cubes and the units from the Base 10 equipment, but not the tens, e.g.  $13 - 4$



Touch count and remove the number to be taken away, in this case 4.



Touch count to find the number that remains.

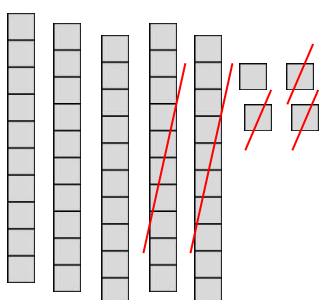


## Y2

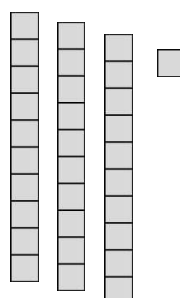
### End of Year Objective:

**Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers.**

Children will begin to use the Base 10 equipment to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate  $54 - 23$ , children would count out 54 using the Base 10 equipment (5 tens and 4 units). They need to consider whether there are enough units/ones to remove 3, in this case there are, so they would remove 3 units and then two tens, counting up the answer of 3 tens and 1 unit to give 31.



which leaves



so  $54 - 23 = 31$

Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks), e.g. to calculate  $39 - 17$  children would draw 39 as 3 tens (lines) and 4 units (dots) and would cross out 7 units and then one ten, counting up the answer of 2 tens and 2 units to give 22.

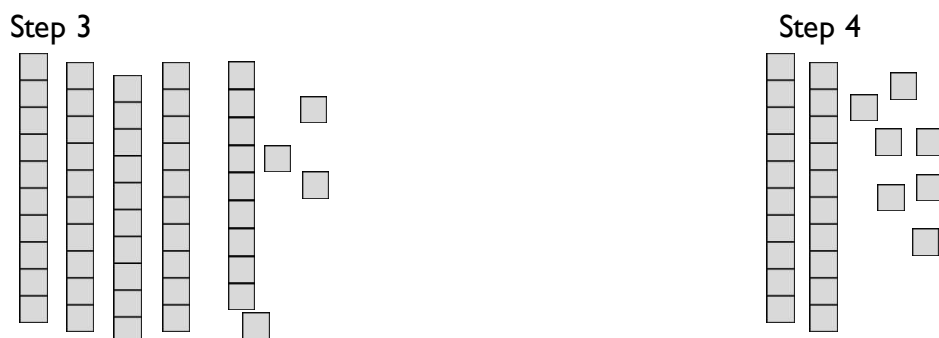


Circling the tens and units that remain will help children to identify how many remain.

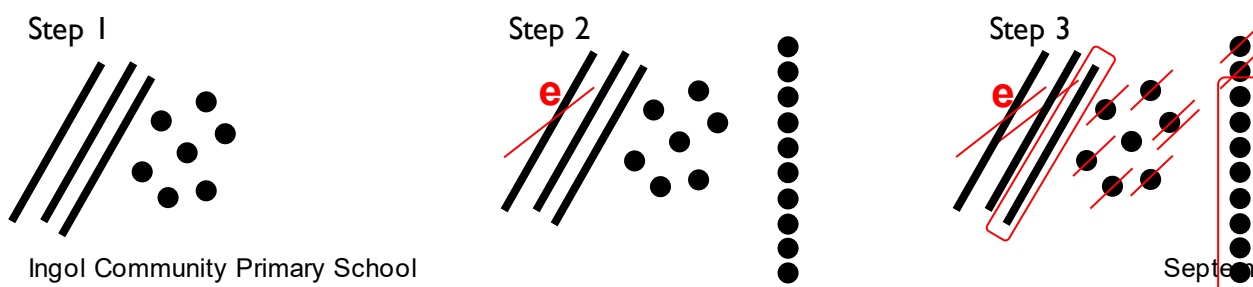
When the amount of units to be subtracted is greater than the units in the original number, an exchange method is required. This relies on children's understanding of ten units being an equivalent amount to one ten. To calculate  $53 - 26$ , by using practical equipment, they would count out 53 using the tens and units, as in Step 1. They need to consider whether there are enough units/ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.



The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.



When recording their own drawings, when calculating  $37 - 19$ , children would cross out a ten and exchange for ten units. The exchanged ten is denoted with an **e** so children recognise this has not been subtracted. Drawing the units in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the units that were already in place.



Circling the tens and units that remain will help children to identify how many remain.

**Y3**

**End of Year Objective:**  
**Subtract numbers with up to three digits, using formal written method of columnar subtraction.\***

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Children will build on their knowledge of using Base 10 equipment from year 2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the units and tens as appropriate (as with the more informal method in year 2).

Step 1

$$\begin{array}{r} 80 \\ - 50 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ 7 \end{array}$$

Step 2

$$\begin{array}{r} 80 \\ - 50 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ 7 \end{array}$$

Step 3


$$\begin{array}{r} 80 \\ - 50 \\ \hline 30 \end{array} \quad \begin{array}{r} 9 \\ 7 \\ 2 \end{array}$$

*Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.*

This will be recorded by the children as:

$$\begin{array}{r} 80 \rightarrow 9 \\ - 50 \rightarrow 7 \\ \hline 30 \rightarrow 2 \end{array} = 32$$


Children can also use jottings of the Base 10 materials (as in year 2) to support with their calculation, as in the example below.



$$\begin{array}{r}
 80 \rightarrow 9 \\
 - 50 \rightarrow 7 \\
 \hline
 30 \rightarrow 2 = 32
 \end{array}$$


From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough units/ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same ( $71 = 70 + 1 = 60 + 11$ ).

Step 1




$$\begin{array}{r}
 70 \\
 - 40 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 1 \\
 - 6 \\
 \hline
 \end{array}$$

Step 2




$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 \end{array}$$

Step 3



$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 \end{array}$$

Step 4



$$\begin{array}{r}
 60 \\
 - 40 \\
 \hline
 20
 \end{array}
 \qquad
 \begin{array}{r}
 11 \\
 - 6 \\
 \hline
 5
 \end{array}$$

This will be recorded by the children as:

$$\begin{array}{r}
 60 \\
 70 \rightarrow 11 \\
 - 40 \rightarrow 6 \\
 \hline
 20 \rightarrow 5 = 25
 \end{array}$$

By the end of year 3, children should also extend this method for three digit numbers.

## Y4

### End of Year Objective:

Subtract numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar subtraction where appropriate.

Children will move to year 4 using whichever method they were using as they transitioned from year 3.

Step 1

$$\begin{array}{r} 700 \rightarrow 50 \rightarrow 4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 2 (exchanging from tens to units)

$$\begin{array}{r} 700 \rightarrow \overset{40}{\cancel{50}} \rightarrow ^14 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 3 (exchanging from hundreds to tens)

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow ^14 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 4

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow ^14 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

This would be recorded by the children as:

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow ^14 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

When children are ready, this leads on to the compact method of decomposition:

$$\begin{array}{r} \phantom{0} 4 \overset{6}{\cancel{5}} \overset{14}{\cancel{5}} \phantom{0} ^14 \\ - 3 \phantom{0} 2 \phantom{0} 8 \phantom{0} 6 \\ \hline \phantom{0} 1 \phantom{0} 4 \phantom{0} 6 \phantom{0} 8 \end{array}$$

By the end of year 4, children should be using the written method confidently and with understanding. They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

## Y5

### End of Year Objective:

**Subtract whole numbers with more than 4 digits *and* decimals with two decimal places, including formal written methods (columnar subtraction).**

Children should continue to use the decomposition method to solve calculations such as:

$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{10}{\phantom{0}} \overset{6}{\cancel{7}} \overset{12}{\phantom{2}} \\ - \phantom{0} 3 \phantom{0} 2 \phantom{0} 2 \phantom{0} 6 \\ \hline 3 \phantom{0} 8 \phantom{0} 4 \phantom{0} 6 \end{array}$$

$$\begin{array}{r} \overset{2}{\cancel{3}} \overset{13}{\phantom{0}} \overset{4}{\cancel{4}} \overset{12}{\phantom{2}} \\ - \phantom{0} 1 \phantom{0} . \phantom{0} 7 \phantom{0} 6 \\ \hline 1 \phantom{0} . \phantom{0} 6 \phantom{0} 6 \end{array}$$

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

## Y6

### End of Year Objective:

**Subtract whole numbers and decimals using formal written methods (columnar subtraction).**

Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

$$\begin{array}{r} \overset{5}{\cancel{6}} \overset{13}{\phantom{0}} \overset{4}{\cancel{4}} \overset{13}{\phantom{0}} \overset{2}{\phantom{2}} \\ - \phantom{0} 4 \phantom{0} 6 \phantom{0} 8 \phantom{0} 1 \\ \hline 1 \phantom{0} 7 \phantom{0} 5 \phantom{0} 1 \end{array}$$

$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{11}{\phantom{0}} \overset{6}{\cancel{7}} \overset{11}{\phantom{0}} \overset{10}{\phantom{0}} \\ - \phantom{0} 3 \phantom{0} 4 \phantom{0} . \phantom{0} 7 \phantom{0} 1 \\ \hline 3 \phantom{0} 8 \phantom{0} 2 \phantom{0} . \phantom{0} 4 \phantom{0} 9 \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

# Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

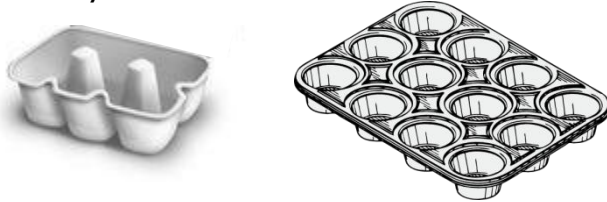
- the inverse of division
- commutative i.e.  $5 \times 3$  is the same as  $3 \times 5$
- associative i.e.  $2 \times 3 \times 5$  is the same as  $2 \times (3 \times 5)$

## **YR**

**Early Learning Goal:**  
**Children solve problems, including doubling.**

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing the fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

## **Y1**

**End of Year Objective:**  
**Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

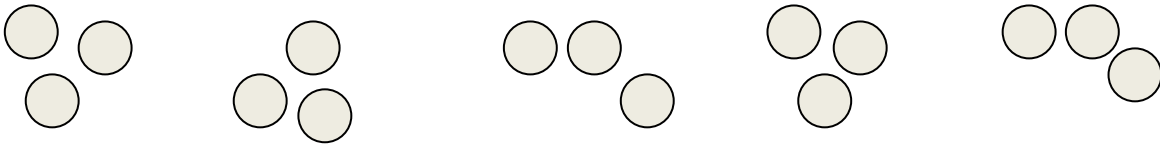
In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

## Y2

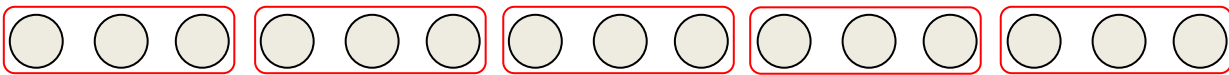
**End of Year Objective:**  
**Calculate mathematical statements for multiplication (using repeated addition) and write them using the multiplication (x) and equals (=) signs.**

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

$5 \times 3$  can be shown as five groups of three with counters, either grouped in a random pattern, as below:

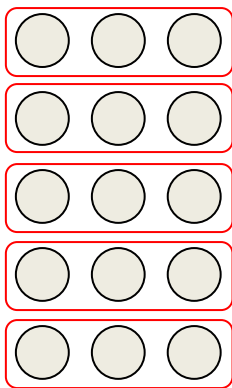


or in a more ordered pattern, with the groups of three indicated by the border outline:

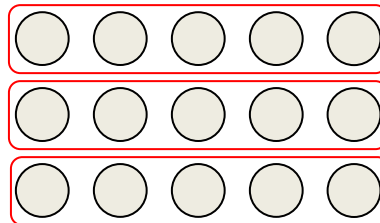


Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

$5 \times 3^*$  can be represented as an array in two forms (as it has commutativity):



$$3 + 3 + 3 + 3 + 3 = 15$$



$$5 + 5 + 5 = 15$$

\*For mathematical accuracy  $5 \times 3$  is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant).

**Y3**

**End of Year Objective:**

**Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.\***

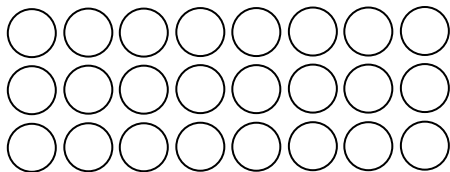
*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$3 \times 8$

They may show this using practical equipment:



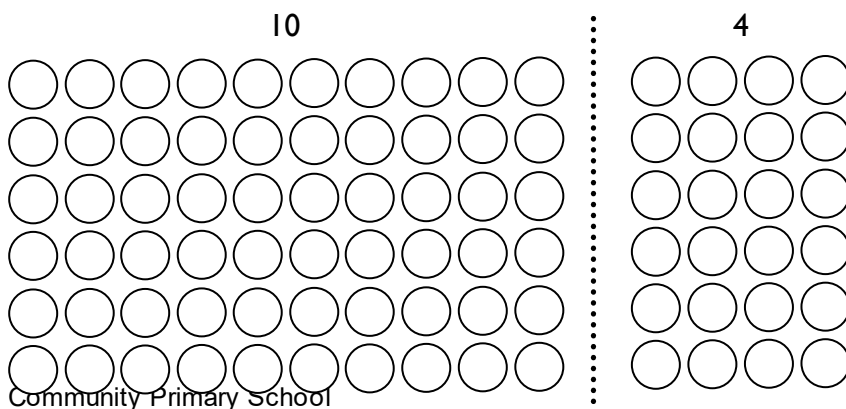
$3 \times 8 = 8 + 8 + 8 = 24$

or by jottings using squared paper:

	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	

$3 \times 8 = 8 + 8 + 8 = 24$

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating  $14 \times 6$ , children should set out the array, then partition the array so that one array has ten columns and the other four.



Partitioning in this way, allows children to identify that the first array shows 10 x 6 and the second array shows 4 x 6. These can then be added to calculate the answer:

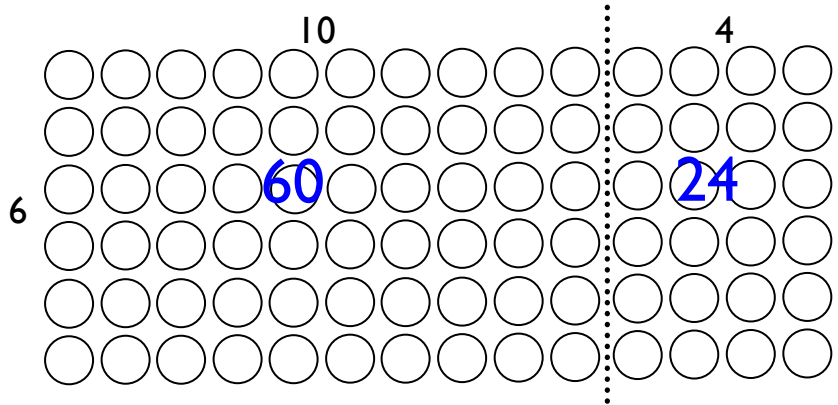
$$(6 \times 10) + (6 \times 4)$$

$$= 60 + 24$$

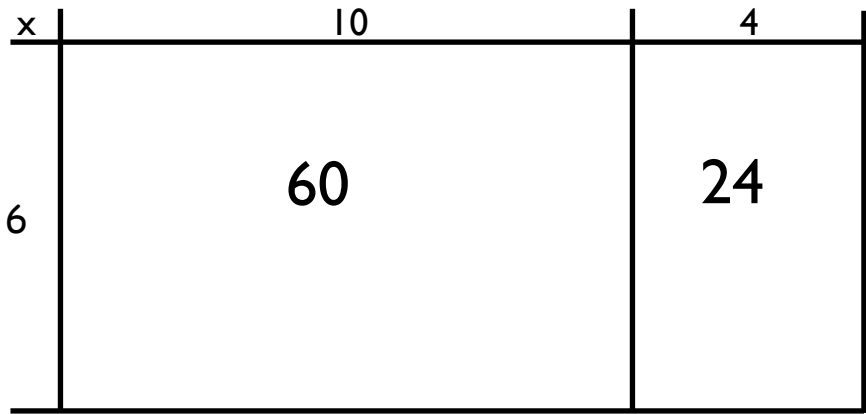
$$= 84$$

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



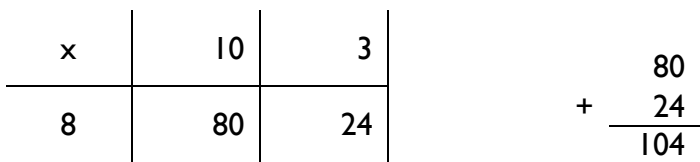
By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

13 x 8



When children are ready, they can then progress to using this method with other two-digit numbers.

$37 \times 6$

x	30	7
6	180	42

$$\begin{array}{r}
 180 \\
 + \quad 42 \\
 \hline
 222
 \end{array}$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

## Y4

### End of Year Objective:

**Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.**

Children will move to year 4 using whichever method they were using as they transitioned from year 3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

$79 \times 8$

x	70	9
8	560	72

$$\begin{array}{r}
 560 \\
 + \quad 72 \\
 \hline
 632
 \end{array}$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that  $7 \times 8 = 56$  to know that  $70 \times 8 = 560$ .

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

$346 \times 8$

x	300	40	6
8	2400	320	48

$$\begin{array}{r}
 2400 \\
 + \quad 320 \\
 + \quad \quad 48 \\
 \hline
 2768
 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

**Y5**

**End of Year Objective:**  
**Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.**

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

4346 x 8

x	4 000	300	40	6
8	32 000	2400	320	48

	32000
+	2400
+	320
+	48
	<hr/>
	34768

and numbers with up to four digits by a two-digit number, e.g.

2693 x 24

x	2000	600	90	3
20	40000	12000	1800	60
4	8000	2400	360	12

	40000
+	8000
+	12000
+	2400
+	1800
+	360
+	60
+	12
	<hr/>
	64632

The long list of numbers in the addition part can be used to check that all of the answers from the grid have been included, however, when children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they should be encouraged to do so. For example,

x	2000	600	90	3
20	40000	12000	1800	60
4	8000	2400	360	12

=	53 860
=	10 772 +
	<hr/>
	64 632

Adding across mentally, leads children to finding the separate answers to:

2 693 x 20

2 693 x 4

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

During Year 5, the transition from the grid method into the formal vertical method for multiplication should take place. The traditional vertical compact method of written multiplication is a highly efficient way to calculate, but it has a very condensed form and needs to be introduced carefully.

It is most effective to begin with the grid method, moving to an expanded vertical layout, before introducing the compact form. This allows children to see, and understand, how the processes relate to each other and where the individual multiplication answers come from e.g.

$$368 \times 6$$

x	300	60	8
6	1 800	360	48

$$\begin{array}{r}
 1800 \\
 + 360 \\
 + 48 \\
 \hline
 2208
 \end{array}$$

Th	H	T	U
	3	6	8
	x 6		
	4	8	(8 x 6)
	3	6	0 (60 x 6)
+	1	8	0 0 (300 x 6)
	2	2	0 8

Th	H	T	U
	3	6	8
	x 6		
	4	8	(8 x 6)
	3	6	0 (60 x 6)
+	1	8	0 0 (300 x 6)
	2	2	0 8

becomes

Th	H	T	U
	3	6	8
	x 6		
	2	2	0 8
	4	4	

The place value columns are labelled to ensure children understand the size of the partitioned digits in the original number(s) and in the answer.

It is vital that the teacher models the correct language when explaining the process of the compact method.

The example shown should be explained as:

*“Starting with the least significant digit... 8 multiplied by 6 is 48, put 8 in the units and carry 4 tens (40). 6 tens multiplied by 6 are 36 tens. Add the 4 tens carried over to give 40 tens (which is the same as 4 hundreds and 0 tens). Put 0 in the tens place of the answer and carry 4 hundreds. 3 hundreds multiplied by 6 are 18 hundreds. Add the 4 hundreds carried over to give 22 hundreds (which is the same as 2 thousands and 2 hundreds). Write 2 in the hundreds place of the answer and 2 in the thousands place of the answer.”*

Children should recognise that the answer is close to an estimated answer of  $400 \times 6 = 2\ 400$

Long multiplication could also be introduced by comparing the grid method with the compact vertical method. Mentally totalling each row of answers is an important step in children making the link between the grid method and the compact method.

x	600	90	3
20	12000	1800	60
4	2400	360	12

$$\begin{array}{r}
 = 13\ 860 \\
 + 2\ 772 \\
 \hline
 16\ 632
 \end{array}$$



$4.92 \times 3$

$$\begin{array}{r} \text{T U . t h} \\ 4.92 \\ \times \quad 3 \\ \hline 0.06 \quad (0.02 \times 3) \\ 2.7 \quad (0.9 \times 3) \\ + 12 \quad (4 \times 3) \\ \hline \underline{14.76} \end{array}$$

becomes

$$\begin{array}{r} \text{T U . t h} \\ 4.92 \\ \times \quad 3 \\ \hline \underline{14.76} \\ 2 \end{array}$$

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

## Progression Towards a Written Method for Division

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of multiplication
- not commutative i.e.  $15 \div 3$  is not the same as  $3 \div 15$
- not associative i.e.  $30 \div (5 \div 2)$  is not the same as  $(30 \div 5) \div 2$

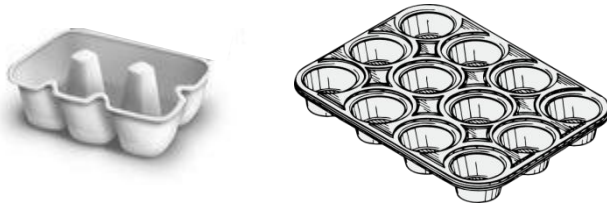
### YR

#### **Early Learning Goal:**

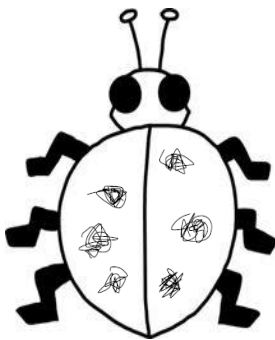
***Children solve problems, including halving and sharing.***

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.



A child's jotting showing how they shared the apples at snack time between two groups.



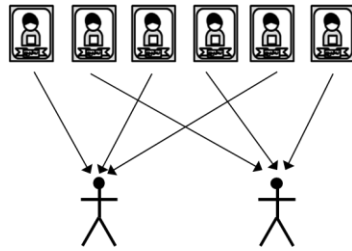
### Y1

#### **End of Year Objective:**

**Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They

may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '... left over'.

## Y2

**End of Year Objective:**  
**Calculate mathematical statements for division within the multiplication tables and write them using the division ( $\div$ ) and equals (=) signs.**

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

$$12 \div 3 =$$

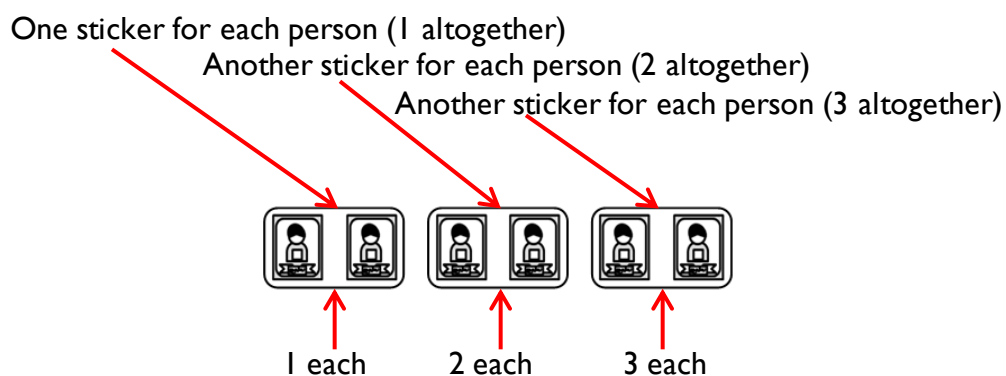


Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

The link between sharing and grouping can be modelled in the following way:

To solve the problem 'If six football stickers are shared between two people, how many do they each get?'

Place the football stickers in a bag or box and ask the children how many stickers would need to be taken out of the box to give each person one sticker each (i.e. 2) and exemplify this by putting the cards in groups of 2 until all cards have been removed from the bag.



Or:

Children should also continue to develop their knowledge of division with remainders, e.g.

$$13 \div 4 =$$



$$13 \div 4 = 3 \text{ remainder } 1$$

Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation  $13 \div 4$ , the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

I have £13. Books are £4 each. How many can I buy?

Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?

Answer: 4 (the remaining 1 apple still need to be placed into a box)

### Y3

**End of Year Objective:**

**Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.\***

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$43 \div 8 =$$



$$43 \div 8 = 5 \text{ remainder } 3$$

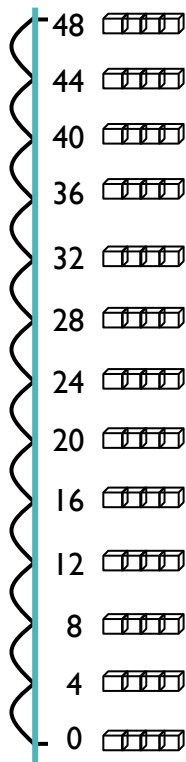
In preparation for developing the ‘chunking’ method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

Stage 1 – repeatedly subtracting individual groups of the divisor

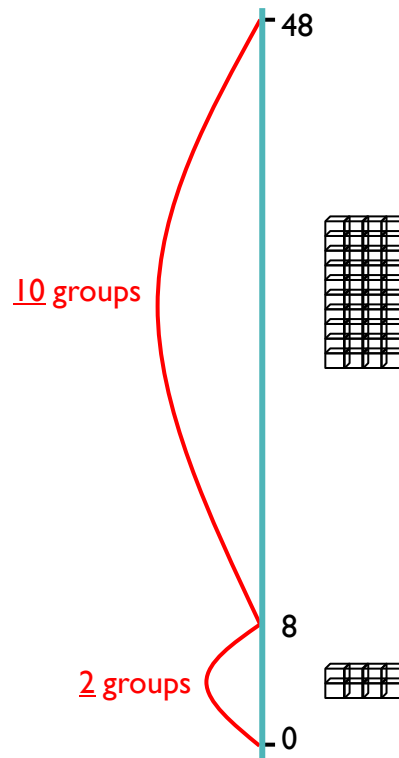
Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

Stage 1  
 $48 \div 4 = 12$  (groups of 4)



Stage 2  
 $48 \div 4 = 10$  (groups of 4) + 2 (groups of 4)  
 $= 12$  (groups of 4)

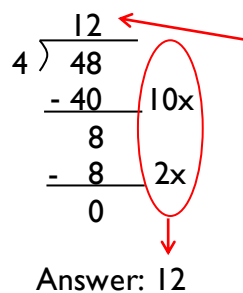
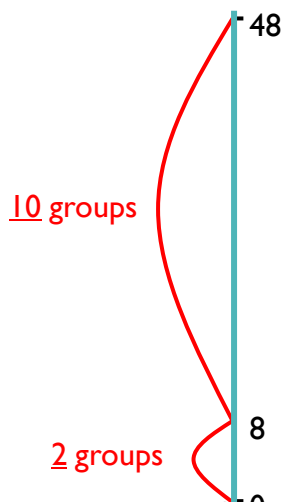


Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

**Y4**

**End of Year Objective:**  
*Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.*

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.



Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

$$73 \div 3$$

$$\begin{array}{r} 24r1 \\ 3 \overline{) 73} \\ - 30 \\ \hline 43 \\ - 30 \\ \hline 13 \\ - 6 \\ \hline 7 \\ - 6 \\ \hline 1 \end{array}$$

Key facts box

1x	3
2x	6
5x	15
10x	30

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

$$196 \div 6$$

$$\begin{array}{r} 32r4 \\ 3 \overline{) 196} \\ - 120 \\ \hline 76 \\ - 60 \\ \hline 16 \\ - 12 \\ \hline 4 \end{array}$$

Key facts box

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

## Y5

### End of Year Objective:

**Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. **During Year 5, children should be encouraged to be efficient when using the chunking method and not have any subtraction steps that repeat a previous step. For example, when performing  $347 \div 8$  an initial subtraction of 160 ( $20 \times 8$ ) and a further subtraction of 160 ( $20 \times 8$ ) should be changed to a single subtraction of 320 ( $40 \times 8$ ).** Also, any remainders should be shown as integers, e.g.

$$523 \div 8$$

$$\begin{array}{r} 65r3 \\ 8 \overline{) 523} \\ - 320 \\ \hline 203 \\ - 160 \\ \hline 43 \\ - 40 \\ \hline 3 \end{array}$$

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x.

$$2458 \div 7$$

$$\begin{array}{r}
 \phantom{7} \overline{) 2458} \text{r}1 \\
 - 2100 \quad 300x \\
 \phantom{7} \underline{358} \\
 - 350 \quad 50x \\
 \phantom{7} \phantom{0} \underline{8} \\
 - 7 \quad 1x \\
 \phantom{7} \phantom{0} \phantom{0} \underline{1}
 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

## Y6

### **End of Year Objective:**

**Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.**

**Use written division methods in cases where the answer has up to two decimal places.**

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

$$6367 \div 28$$

$$\begin{array}{r}
 \phantom{28} \overline{) 6367} \text{r}11 \\
 - 5600 \quad 200x \\
 \phantom{28} \phantom{0} \underline{767} \\
 - 560 \quad 20x \\
 \phantom{28} \phantom{0} \phantom{0} \underline{207} \\
 - 140 \quad 5x \\
 \phantom{28} \phantom{0} \phantom{0} \phantom{0} \underline{67} \\
 - 56 \quad 2x \\
 \phantom{28} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \underline{11}
 \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places.

This should first be demonstrated using a simple calculation such as  $13 \div 4$  to show the remainder initially as a fraction.



Using practical equipment, children can see that for  $13 \div 4$ , the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is  $\frac{1}{4}$ . To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

$$3574 \div 8$$

$$\begin{array}{r} 8 \overline{) 3574} \\ - 3200 \\ \hline 374 \\ - 320 \\ \hline 54 \\ - 48 \\ \hline 6 \end{array}$$

400x  
40x  
6x

$$\frac{6}{8} \leftarrow \begin{array}{l} \text{remainder} \\ \text{divisor} \end{array}$$

So  $3574 \div 8$  is  $446\frac{6}{8}$   
(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half:  $\frac{1}{2} = 0.5$

Quarters:  $\frac{1}{4} = 0.25$ ,  $\frac{3}{4} = 0.75$

Fifths:  $\frac{1}{5} = 0.2$ ,  $\frac{2}{5} = 0.4$ ,  $\frac{3}{5} = 0.6$ ,  $\frac{4}{5} = 0.8$

Tenths:  $\frac{1}{10} = 0.1$ ,  $\frac{2}{10} = 0.2$ ,  $\frac{3}{10} = 0.3$ ,  $\frac{4}{10} = 0.4$ ,  $\frac{5}{10} = 0.5$ ,  $\frac{6}{10} = 0.6$ ,  $\frac{7}{10} = 0.7$ ,  $\frac{8}{10} = 0.8$ ,  $\frac{9}{10} = 0.9$

and reduce other equivalent fractions to their lowest terms.

In the example above,  $3574 \div 8$ , children should be able to identify that the remainder as a fraction of  $\frac{6}{8}$  can be written as  $\frac{3}{4}$  in its lowest terms. As  $\frac{3}{4}$  is equivalent to 0.75, the answer can therefore be written as 446.75.

# Progression Toward Mental Calculation Strategies

## (Addition and Subtraction)

The ability to calculate mentally is an essential skill, but, as with written methods of calculation, children need to be taught. **It is important to ensure that when teaching particular strategies, children have the appropriate prerequisite skills and are guided as to how and when that strategy is appropriate.**

Children should be taught and encouraged to ask themselves the following questions when faced with a calculation:

- Do I know the answer?
- Can I work it out in my head?
- Do I need to do a jotting?
- Do I need to use a written method?

When using a jotting, there is no requirement to follow a particular method of recording.

A feature of mental calculation is that a type of calculation can often be worked out in several different ways. Which method is best will depend on the numbers involved, the age of the children and the range of methods that they are confident with.

In developing a progression through mental calculation strategies for addition and subtraction, it is important that children understand the relevant concepts, in that addition is:

- combining two or more groups to give a total or sum
- increasing an amount

and subtraction is:

- removal of an amount from a larger group (take away)
- comparison of two amounts (difference)

They also need to understand and work with certain principles, that:

- addition and subtraction are inverses
- addition is commutative i.e.  $5 + 3 = 3 + 5$  but subtraction is not  $5 - 3$  is not the same as  $3 - 5$
- addition is associative i.e.  $5 + 3 + 7 = 5 + (3 + 7)$  but subtraction is not  $10 - 3 - 2$  is not the same as  $10 - (3 - 2)$

Commutativity and associativity mean that calculations can be rearranged, e.g.  $4 + 13 = 17$  is the same as  $13 + 4 = 17$ .

## **YR**

### **Early Learning Goal:**

***Using quantities and objects, children add and subtract two single digit numbers and count on or back to find the answer.***

In the EYFS, children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

To solve addition and subtraction problems, they may know familiar calculations such as  $5 + 5$  or  $10 - 1$ , but for other calculations, they may use either a counting all or counting on strategy for addition and a taking away strategy for subtraction. They will subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left.

Counting forwards is the preferable strategy at this stage as counting back requires an abstract understanding of the number system.

## **Y1**

### **End of Year Objective:**

***Add and subtract one-digit and two-digit numbers to 20, including zero.***

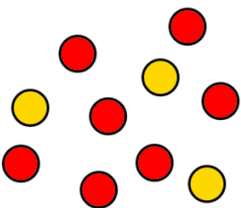
### **Rapid Recall**

Children should be able to:

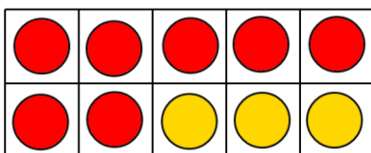
- represent and use number bonds and related subtraction facts within 20

Number bonds can be represented practically using:

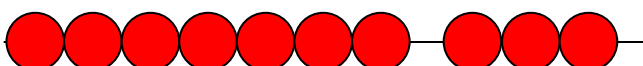
Double sided counters:



Ten frames:



Beadstrings



## **Mental Strategies**

### ***Count on or back in ones (chain count and linked to objects, i.e. 1-1 correspondence)***

Initially, children's counting for addition and subtraction should be linked to the objects that they are using to represent the calculation, e.g. cubes, counters etc. It is important that at this stage the counting and calculating are supported by practical equipment and/or be in context so that they support children's developing understanding of the concepts of addition and subtraction in a concrete rather than abstract way.

Children can begin to use chain counting (i.e. unsupported by objects) when they are confident with the concepts of addition and subtraction and have developed their understanding of using counting on or back, rather than counting all, as a strategy for these calculations.

*Examples of calculations:*

$4 + 5$	count on in ones from 4 (or in ones from 5)
$8 - 3$	count back in ones from 8
$10 + 7$	count on in ones from 10 (or use place value)
$13 + 5$	count on in ones from 13
$17 - 3$	count back in ones from 17

*Prerequisite skills:*

- Count using one to one correspondence
- Count forwards and backwards in ones

To develop an understanding of addition and subtraction, the progression through learning should be:

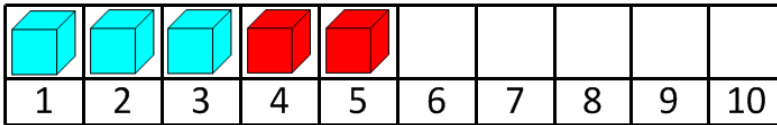
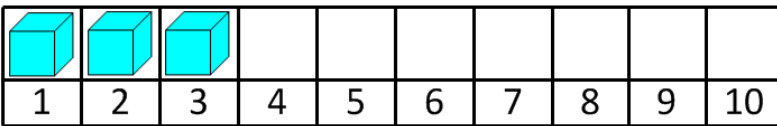
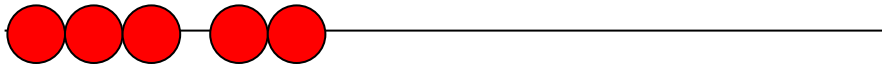
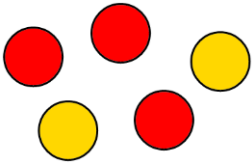
Concrete  $\longrightarrow$  Model  $\longrightarrow$  Abstract

An example of this might be:

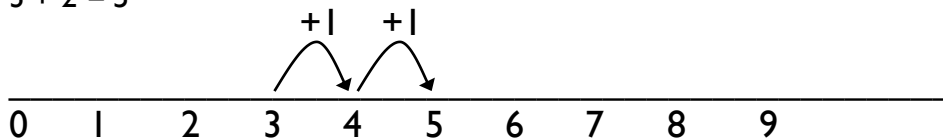
Using counters  $\longrightarrow$  Using a beadstring  $\longrightarrow$  Placing cubes on a number track  $\longrightarrow$  Using a numberline  
(Concrete – random) (Concrete – organised) (Model) (Abstract)

## Addition

$3 + 2 = 5$



$3 + 2 = 5$



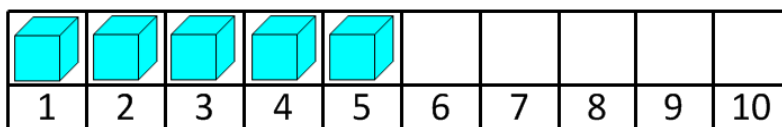
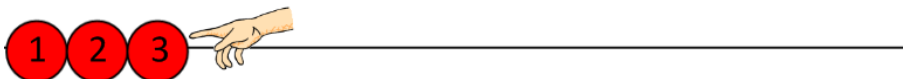
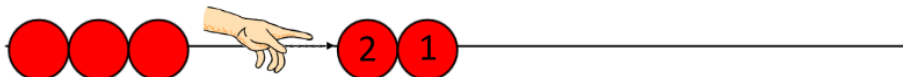
## Subtraction

$5 - 2 = 3$

Touch count and remove the number to be taken away, in this case 2.



Touch count to find the number that remains.



Counters

Beadstring

Number track stage 1

Number track stage 2

Numberline

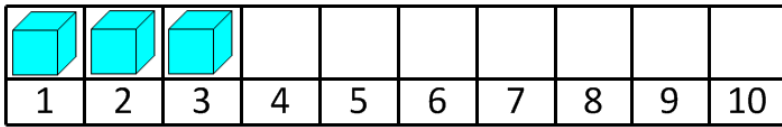
Counters stage 1

Counters stage 2

Beadstring stage 1

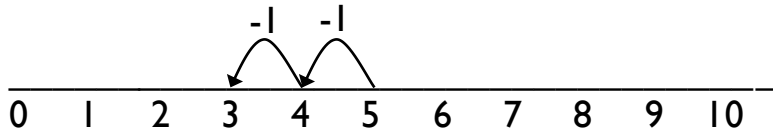
Beadstring stage 2

Number track stage 1



Number track stage 2

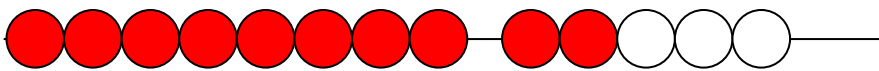
$$5 - 2 = 3$$



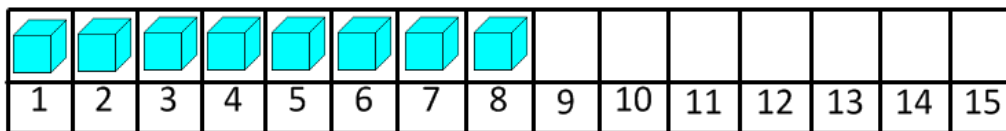
Numberline

**Addition**

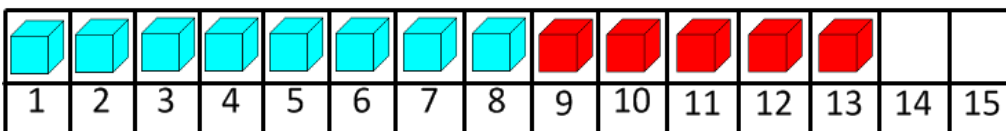
$$8 + 5 = 13$$



Beadstring

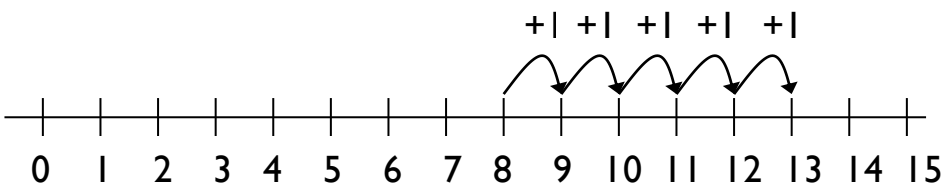


Number track stage 1



Number track stage 2

$$8 + 5 = 13$$

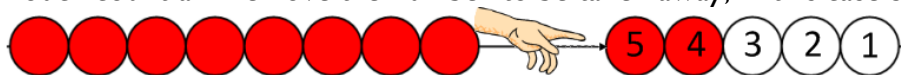


Numberline

**Subtraction**

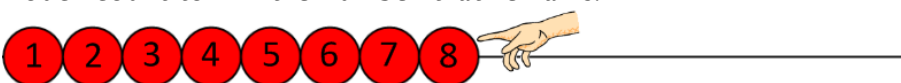
$$13 - 5 = 8$$

Touch count and remove the number to be taken away, in this case 5.

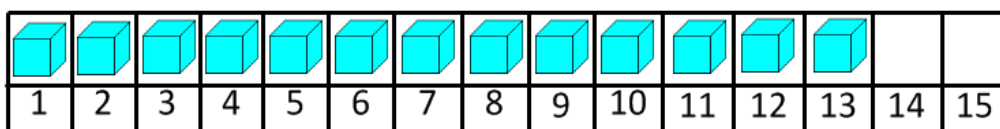


Beadstring stage 1

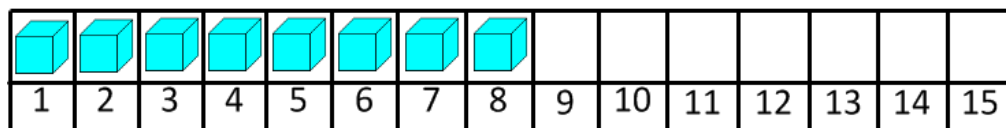
Touch count to find the number that remains.



Beadstring stage 2

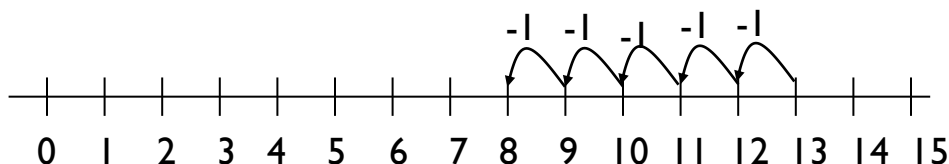


Number track stage 1



Numbertrack stage 2

$$13 - 5 = 8$$



Numberline

### Reorder numbers in a calculation

In Y1, children need to recognise that they can rearrange an addition, but not a subtraction. They also need to understand that the principle behind reordering a calculation is to make it more efficient, particularly when utilising a counting on strategy. Children need to be encouraged to identify calculations which should be reordered and those that are already in the most efficient format.

Examples of calculations:

- |          |  |
|----------|--|
| $8 + 3$  | doesn't need reordering as the greater number is first already |
| $2 + 7$  | reorder as $7 + 2$   |
| $5 + 13$ | reorder as $13 + 5$  |
| $11 + 6$ | doesn't need reordering as the greater number is first already |

Prerequisite skills:

- Understand the place value of numbers to identify which number is the greater
- Understand that reordering works (at this stage) for addition but not subtraction\* (*because children are not at the level when they are solving calculations such as  $16 - 3 - 6$ , when reordering would be appropriate*).

### Partition small numbers, e.g. $8 + 3 = 8 + 2 + 1$

Utilising partitioning in this way is useful as a strategy for bridging across 10 or multiples of 10 to make calculations more efficient.

Examples of calculations:

- |         |                            |
|---------|----------------------------|
| $7 + 5$ | partitioned as $7 + 3 + 2$ |
| $9 + 7$ | partitioned as $9 + 1 + 6$ |
| $6 + 8$ | partitioned as $6 + 4 + 4$ |

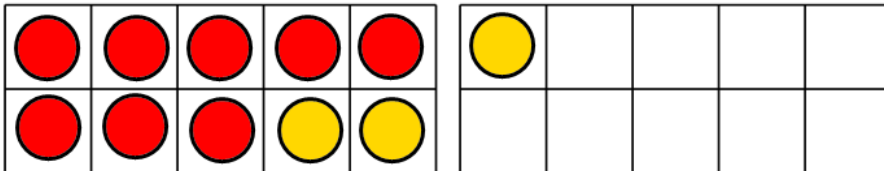
Prerequisite skills:

- Partition numbers in different ways, e.g. 5 as  $2 + 3$  to enable  $8 + 5$  as  $8 + 2 + 3$
- Know, or quickly derive, number bonds for numbers up to and including 10

This method can be supported by the use of practical equipment, e.g.

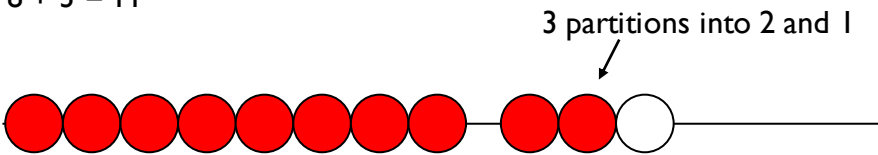
**Addition**

$8 + 3 = 11$



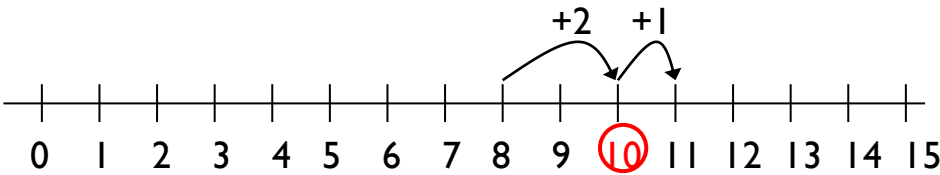
Ten frame

$8 + 3 = 11$



Beadstring

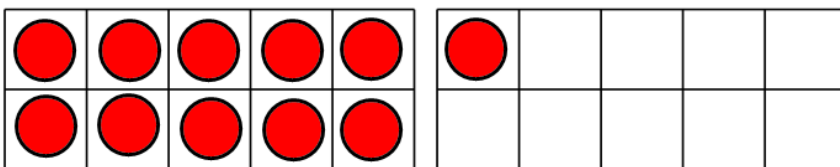
$8 + 3 = 11$



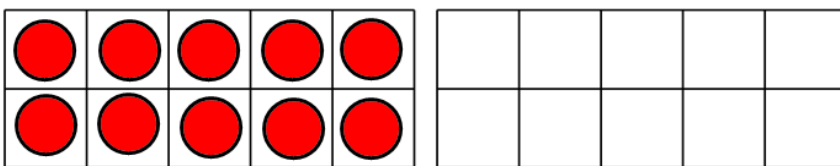
Numberline

**Subtraction**

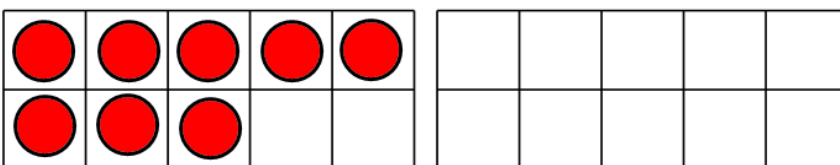
$11 - 3 = 8$



Ten frame stage 1

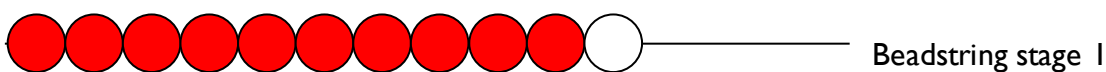


Ten frame stage 2 (take away 1)

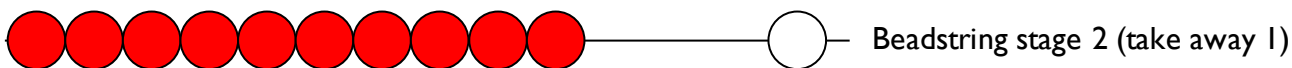


Ten frame stage 3 (take away 2)

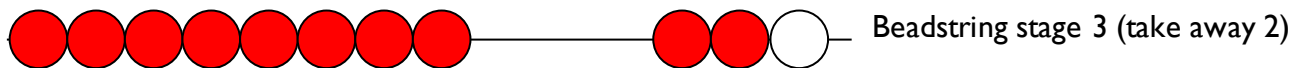
$$11 - 3 = 8$$



Beadstring stage 1

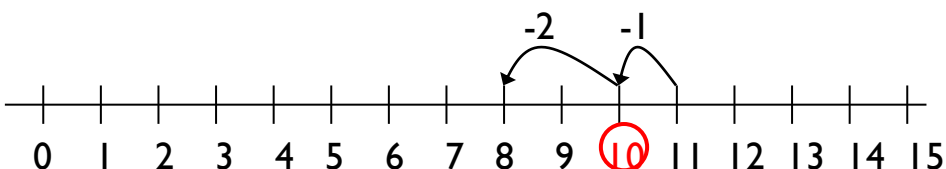


Beadstring stage 2 (take away 1)



Beadstring stage 3 (take away 2)

$$11 - 3 = 8$$



## Y2

### End of Year Objective:

*Add and subtract numbers mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers.*

### Rapid Recall

Children should be able to:

- recall and use addition and subtraction facts to 20 fluently
- derive and use related facts up to 100

Beadstrings are useful for deriving and using related facts up to 100.

$$60 + 40 = 100$$



### Mental Strategies

#### Partition and combine multiples of tens and ones

Partitioning numbers is a core strategy for adding and subtracting pairs of numbers. Children can either partition both of the numbers in the calculation, or keep the first number the same and just partition the second. They should be encouraged to use mental methods when adding or subtracting:

- multiples of 10
- TU + or – U (not crossing tens boundaries)
- TU + or – TU (not crossing tens boundaries)

*Examples of calculations*

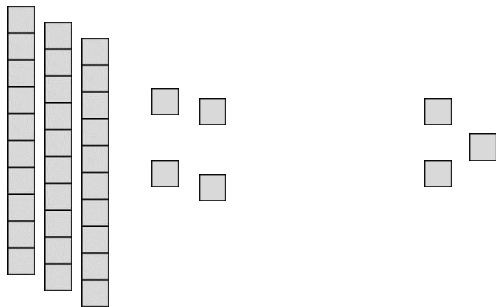
$40 + 37$        $40$  add  $30$  and  $7 = 40$  add  $30$  add  $7$   
 $15 + 14$        $10$  and  $5$  add  $10$  and  $4 = 10$  add  $10$  add  $5$  add  $4$  or  $15$  add  $10$  add  $4$   
 $37 + 12$        $37$  add  $10$  and  $2 = 37$  add  $10$  add  $2$   
 $78 - 42$        $78$  take away  $40$  and  $2 = 78$  take away  $40$  take away  $2$   
 $80 - 35$        $80$  take away  $30$  and  $5 = 80$  take away  $30$  take away  $5$

*Prerequisite skills:*

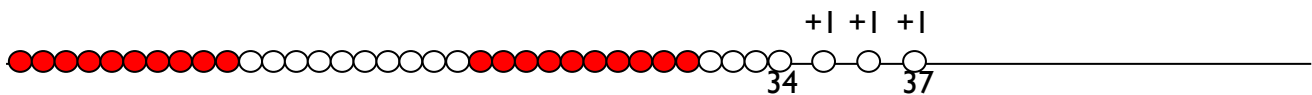
- Count using one to one correspondence
- Count forwards and backwards in ones and tens from any one- or two-digit number
- Understand place value, understand which digit represents tens and which digit represents ones and identify what changes if one is added or subtracted, and what changes if ten is added or subtracted.
- Partition numbers into tens and ones

**Addition**

$34 + 3 = 37$  (shown using Base 10 equipment)



$34 + 3 = 37$  (shown using a beadstring)



$34 + 3 = 37$  (shown using a numberline)

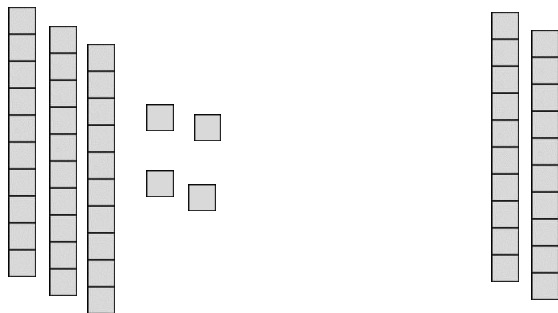


$34 + 20 = 54$  (shown using Base 10 equipment)

Children could use Base 10 equipment to calculate this as:

$$30 + 20 = 50$$

$$50 + 4 = 54$$

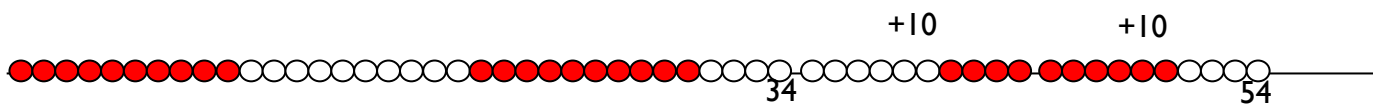


$34 + 20 = 54$  (shown using a beadstring)

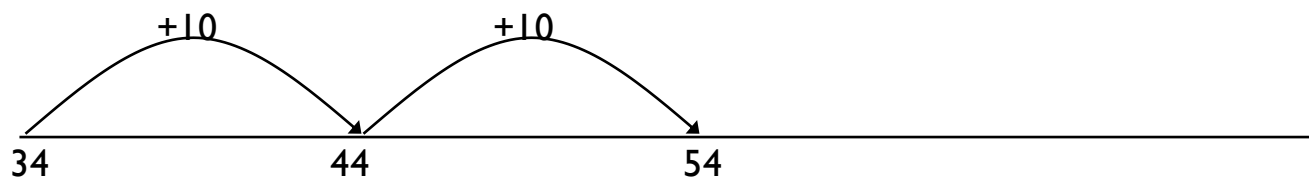
Children could use a beadstring to calculate this as:

$$34 + 10 = 44$$

$$44 + 10 = 54$$



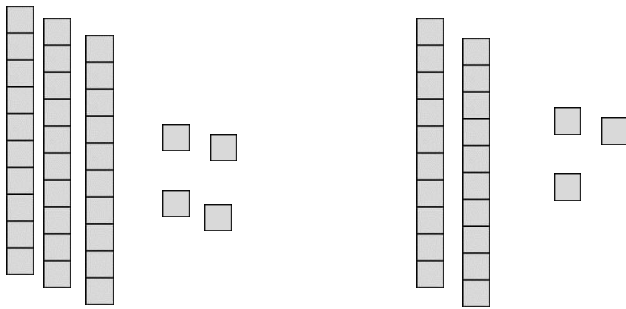
$34 + 20 = 54$  (shown using a numberline)



$34 + 23 = 57$  (shown using Base 10 equipment to partition both numbers)

Children could use Base 10 equipment to calculate this as:

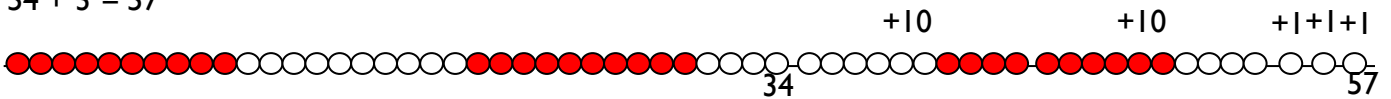
$30 + 20 = 50$   
 $4 + 3 = 7$   
 $50 + 7 = 57$



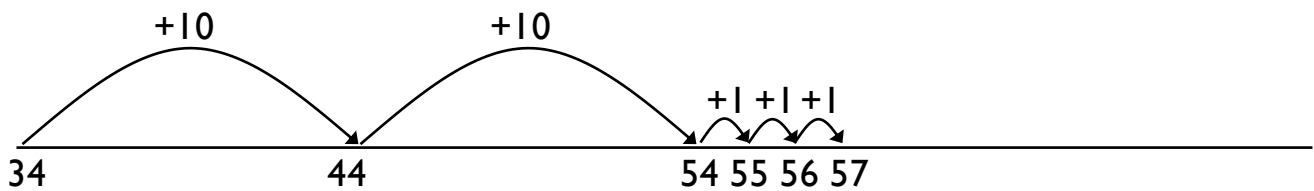
$34 + 23 = 57$  (shown using a beadstring to keep the first number the same and just partition the second)

Children could use a beadstring to calculate this as:

$34 + 10 = 44$   
 $44 + 10 = 54$   
 $54 + 3 = 57$

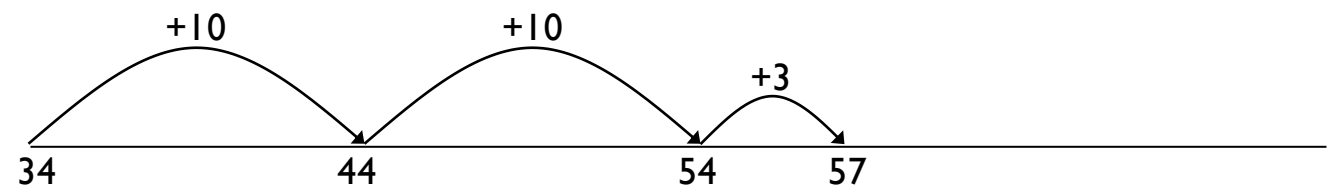


$34 + 23 = 57$  (shown using a numberline to keep the first number the same and just partition the second)



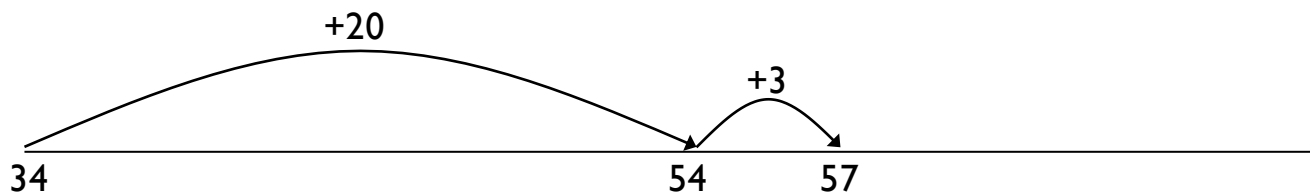
Encourage children to become more efficient by adding the units in one jump (by using the known fact  $4 + 3 = 7$ ).

$34 + 23 = 57$



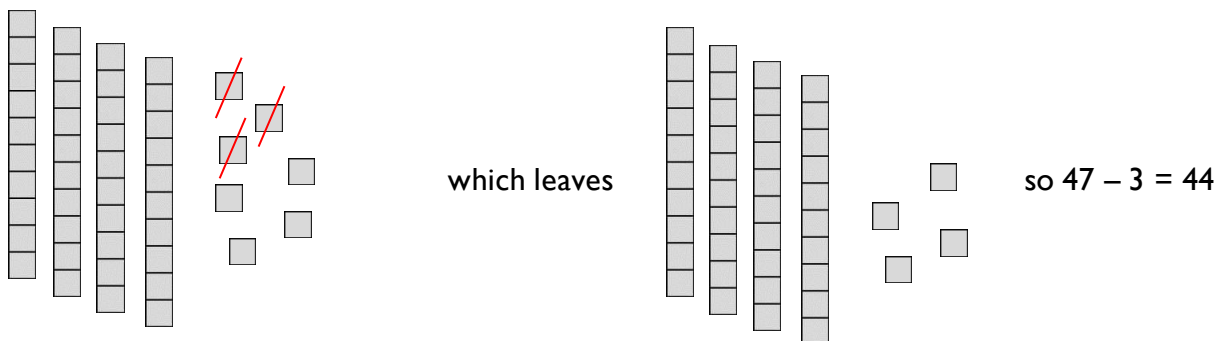
Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$

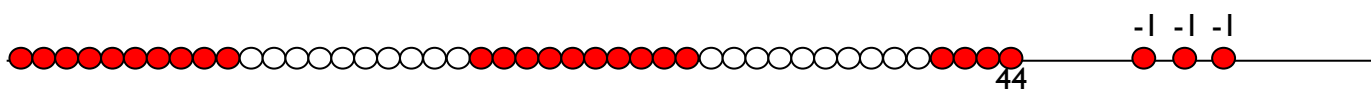


### Subtraction

$$47 - 3 = 44 \text{ (shown using Base 10 equipment)}$$



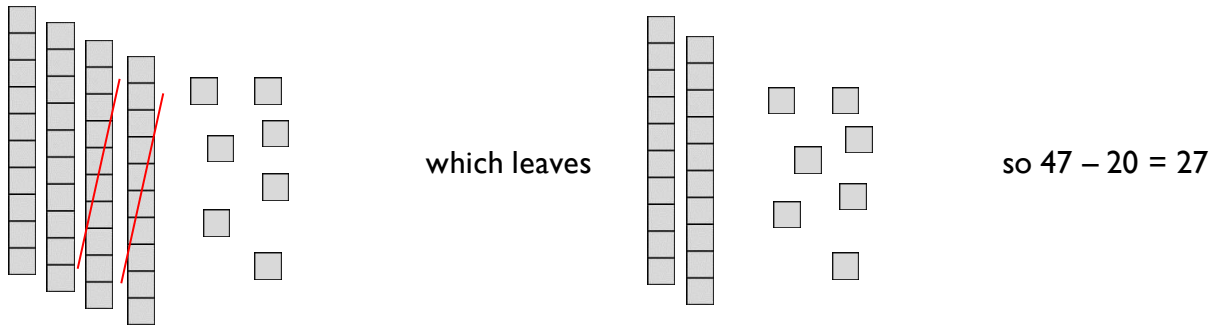
$$47 - 3 = 44 \text{ (shown using a beadstring)}$$



$$47 - 3 = 44 \text{ (shown using a numberline)}$$



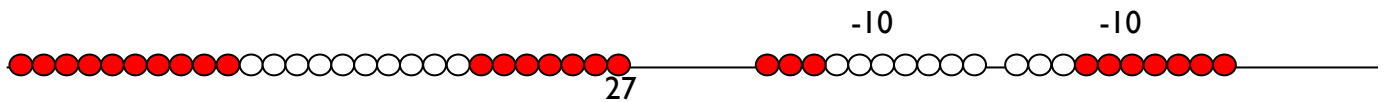
$47 - 20 = 27$  (shown using Base 10 equipment)



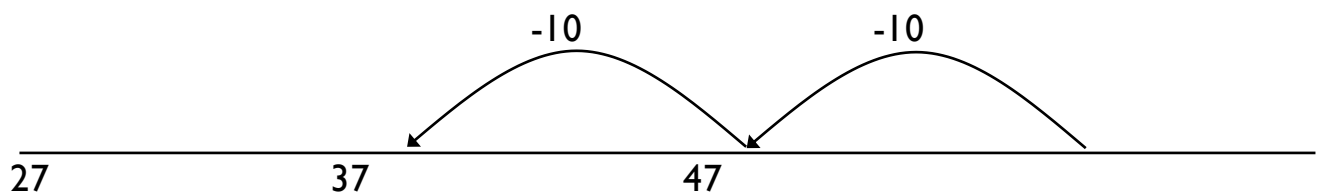
$47 - 20 = 27$  (shown using a beadstring)

Children could use a beadstring to calculate this as:

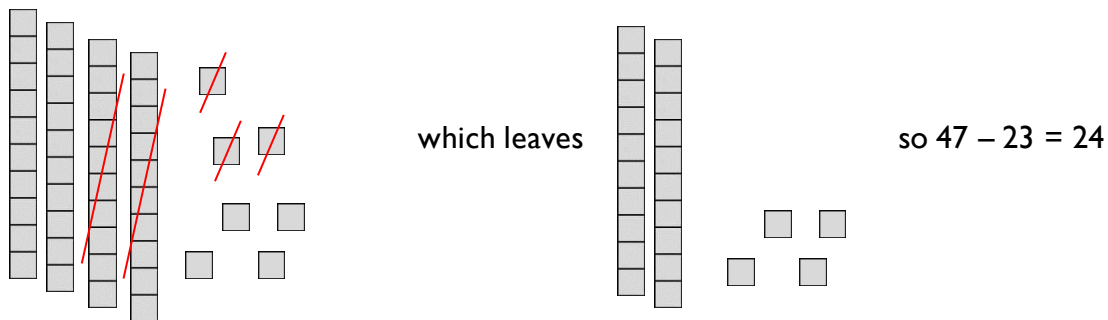
$$47 - 10 = 37$$
$$37 - 10 = 27$$



$47 - 20 = 27$  (shown using a numberline)



$47 - 23 = 24$  (shown using Base 10 equipment)



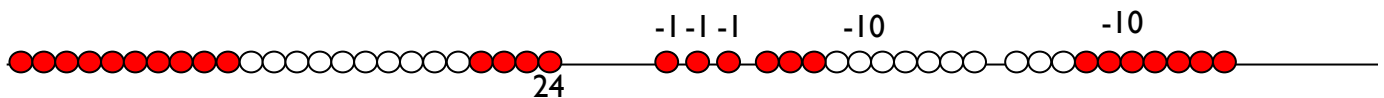
$47 - 23 = 24$  (shown using a beadstring)

Children could use a beadstring to calculate this as:

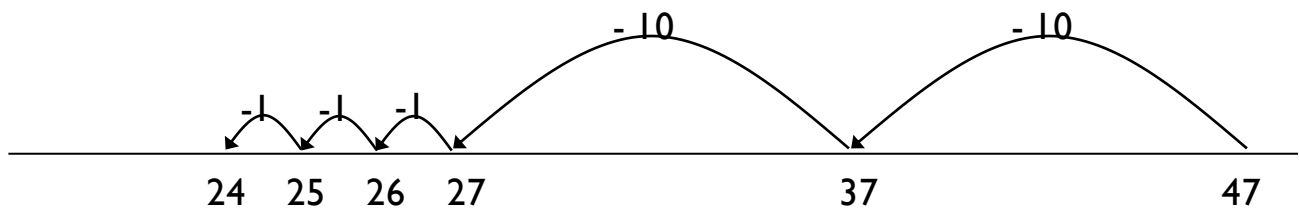
$$47 - 10 = 37$$

$$37 - 10 = 27$$

$$27 - 3 = 24$$

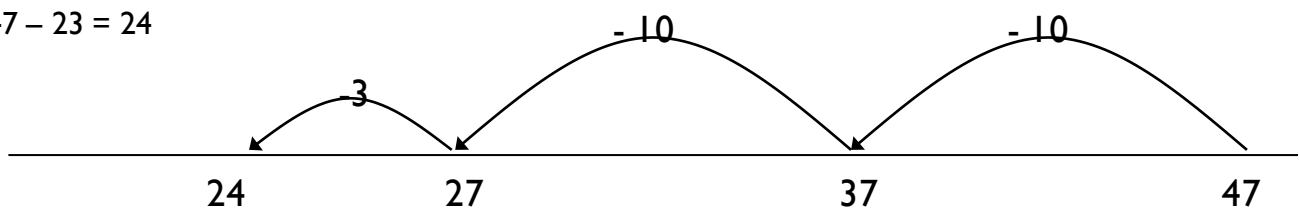


$47 - 23 = 24$  (shown using a numberline)



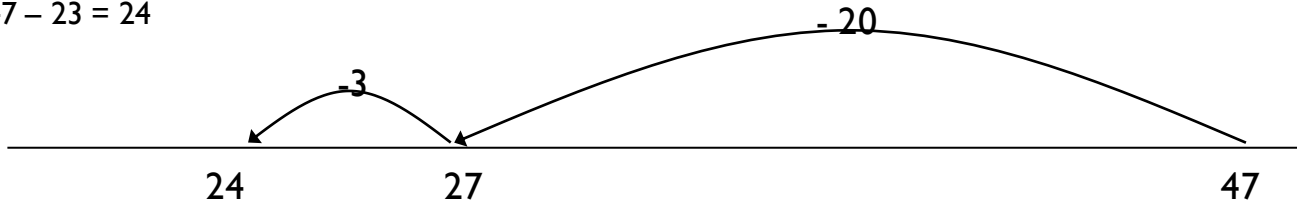
Encourage children to become more efficient by subtracting the units in one jump (by using the known fact  $7 - 3 = 4$ ).

$$47 - 23 = 24$$



Followed by subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



### Reorder numbers in a calculation

In Y2, children need to recognise that they can rearrange an addition, but not a subtraction. They also need to understand that the principle behind reordering a calculation is to make it more efficient, particularly when utilising a counting on strategy. Children need to be encouraged to identify calculations which should be reordered and those that are already in the most efficient format. When adding three single digit numbers, reordering should be based on number bonds or doubles with which the child is familiar.

Examples of calculations:

- 5 + 34      34 + 5
- 42 + 11      doesn't need reordering as the greater number is first already
- 5 + 7 + 5      5 + 5 + 7 (utilising knowledge of number bonds or doubles)

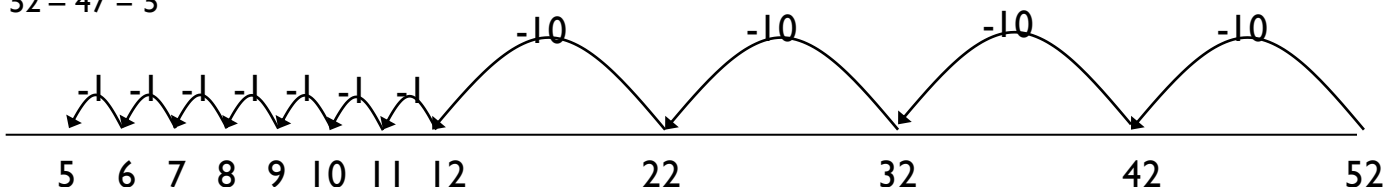
Prerequisite skills:

- Understand the place value of numbers to identify which number is the greater
- Understand that reordering works for addition but not subtraction\* (because children are not at the level when they are solving calculations such as  $16 - 3 - 6$ , when reordering would be appropriate).

### Find a small difference by counting up from the lesser to the greater number

Children should, using their knowledge of place value, be able to identify when numbers are close together. When that is the case, it is more efficient, when subtracting, to count on to find the difference, rather than taking away. For example, in the calculation  $52 - 47$ , to solve this by:

$$52 - 47 = 5$$



is far less efficient than:



47 48 49 50 51 52 (counting up from 47 – 52)

For children to use this method with understanding, it is important that they understand how counting on links to subtraction.

Initially, they should look at simple numbers to develop an understanding of the concept of difference and counting on. For example, with  $7 - 4$ , they can make two towers, one of 7 cubes and one of 4 cubes (Step 1). The calculation can be phrased as ‘How many more do we need to make the towers the same size?’ To answer this question, the children can add cubes of a different colour onto the smaller tower until they are the same height.

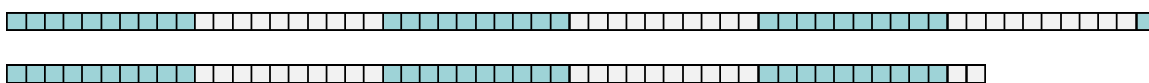


3 more cubes are needed to make them the same size, so the difference between 7 and 4 is 3. This could be compared to taking away 4 from 7 so that children can see that it is the same answer.

The next stage from this would be to encourage children to use the cubes to make lines rather than towers.



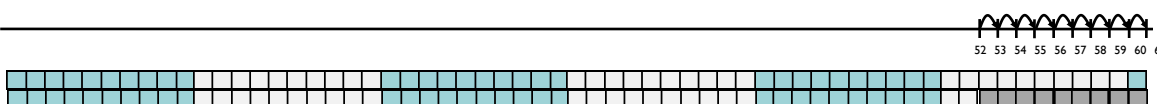
Once children can find the difference using this method using numbers up to 20, they can continue to use this strategy to solve calculations with two-digit numbers, using base 10 materials rather than cubes. For example, with the calculation  $61 - 52$ , children can use base 10 to set out two lines, one for each number (the base 10 in the illustration are two colours to enable tens to be identified, this does not need to be the case with the materials children are using).



To find how many more are needed, or the difference, children would use a second colour of base ten ones to make the lines the same:



To make this a more sustainable method, it can be modelled alongside a number line jotting, e.g.



Examples of calculations

$52 - 47$

$74 - 66$

$81 - 79$

$32 - 25$

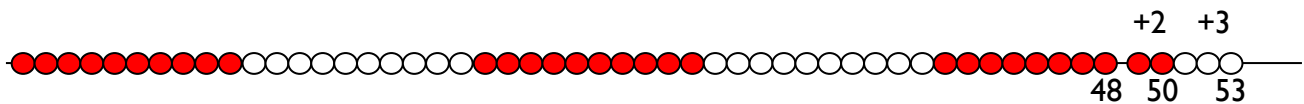
Prerequisite skills:

- Understand the place value of numbers to identify which number is the greater or lesser
- Place numbers on a partially marked and then unmarked number line
- Establish whether numbers are close together
- Count forwards and backwards in ones and tens from any one- or two-digit number

**Begin to bridge through 10 when adding a single digit number (partitioning, e.g.  $58 + 5 = 58 + 2 + 3$ )**

Use of the bridging strategy relies heavily on children's efficient and accurate recall of number bonds to 10 or how far away a number is from a multiple of 10 (see use of 10 frames in Year 1). When calculating, e.g.  $48 + 5$ , consider using bead strings or different coloured blocks of 10 cubes to illustrate it as  $48 + 2 + 3$  using the natural colour demarcations in the bead string to support this identification. This can also be shown using 10 frames (see Year 1 'Partition small numbers' section for more information).

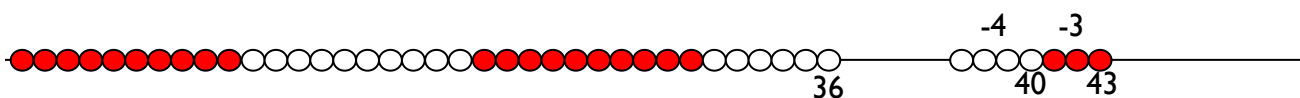
$48 + 5 = 53$



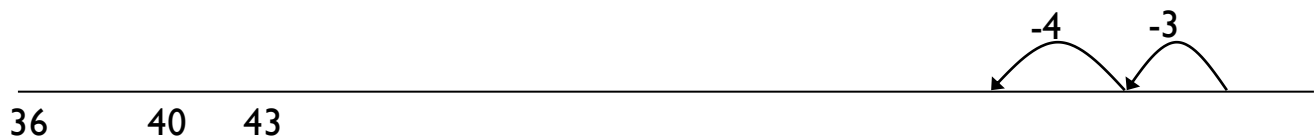
$48 + 5 = 53$



$43 - 7 = 36$



$43 - 7 = 36$



### Examples of calculations

$$\begin{array}{ll} 25 + 6 & \text{as } 25 + 5 + 1 \\ 12 - 7 & \text{as } 12 - 2 - 5 \\ 66 + 7 & \text{as } 66 + 4 + 3 \\ 43 - 7 & \text{as } 43 - 3 - 4 \end{array}$$

### Prerequisite skills:

- Partition numbers in different ways, e.g. 5 as 2 + 3 to enable  $58 + 5$  as  $58 + 2 + 3$
- Know, or quickly derive, number bonds to 10

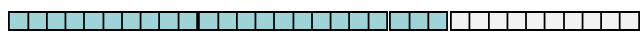
### Add or subtract 9 and 19 by rounding and compensating

Children need to understand both the number system and number bonds in order to understand how to use a compensation method.

For adding 9, children should be shown how to add nine by using base 10 materials and then add ten to the same number to identify what would need to be adjusted to make the calculation correct, e.g.  $23 + 9$



$23 + 9$



$23 + 10$  is one too many, so I have to subtract one

*NB Teaching children to add nine on a hundred square without developing their understanding will not support their ability to understand and use this method effectively.*

### Examples of calculations

$$\begin{array}{ll} 34 + 9 & \text{as } 34 + 10 - 1 \\ 77 + 19 & \text{as } 77 + 20 - 1, \text{ or } 77 + 10 + 10 - 1 \\ 46 - 9 & \text{as } 46 - 10 + 1 \\ 63 - 19 & \text{as } 63 - 20 + 1, \text{ or } 63 - 10 - 10 + 1 \end{array}$$

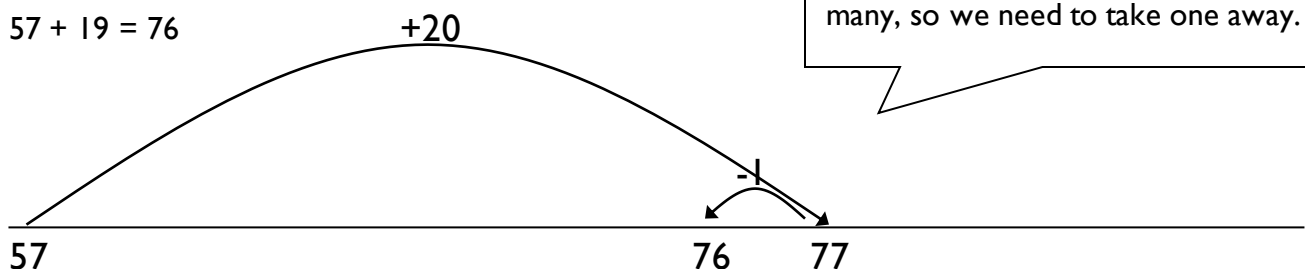
### Prerequisite skills:

- Understand the relationship between 9 and 10 (i.e. a difference of 1)
- Be able to show visually using base 10 equipment

Empty numberlines could be used to model the calculation.

### Addition

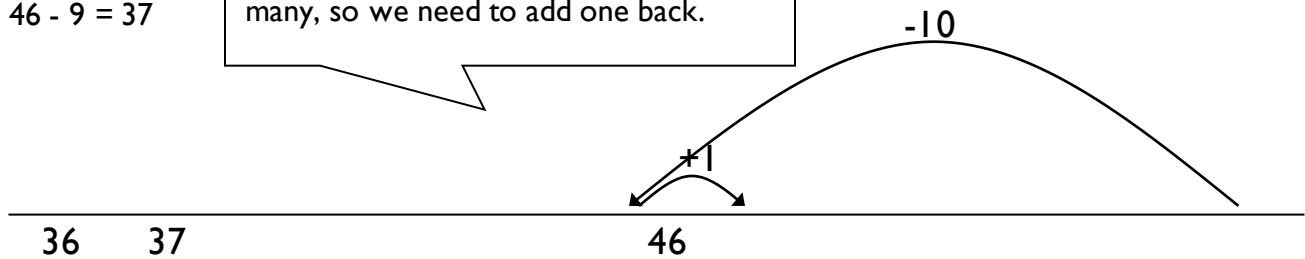
$57 + 19 = 76$



## **Subtraction**

$$46 - 9 = 37$$

We've subtracted ten which is one too many, so we need to add one back.



## **Y3**

### **End of Year Objective:**

**Add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds.**

## **Rapid Recall**

Children should be able to:

- recall and use addition and subtraction facts for 100 (multiples of 5 and 10)
- derive and use addition and subtraction facts for 100
- derive and use addition and subtraction facts for multiples of 100 that total 1000

## **Mental Strategies**

### ***Partition and combine multiples of hundreds, tens and ones***

Partitioning numbers is a core strategy for adding and subtracting pairs of numbers. Children can either partition both of the numbers in the calculation, or keep the first number the same and just partition the second. (See Y2 for more information).

*Examples of calculations:*

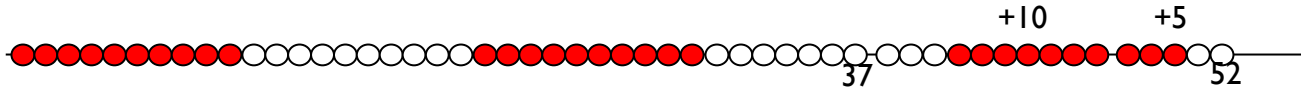
526 + 200	counting on in hundreds
137 + 40	counting on in tens
272 + 8	counting on in ones
428 - 200	counting back in hundreds
323 - 70	counting back in tens
693 - 8	counting back in ones
37 + 15	37 add 10 and 5 = 37 add 10 add 5 (crossing tens boundaries)
42 - 25	42 take away 20 and 5 = 42 take away 20 take away 5 (crossing tens boundaries)

*Prerequisite skills:*

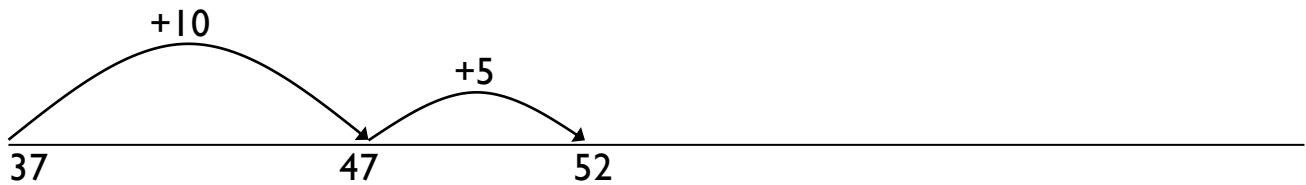
- Count forwards and backwards in ones, tens and hundreds from any one-, two- or three-digit number
- Understand place value and understand which digit changes if one, ten or hundred is added or subtracted
- Partition numbers into hundreds, tens and ones

## Addition

$37 + 15 = 52$  (shown using a beadstring)



$37 + 15 = 52$  (shown using a numberline)



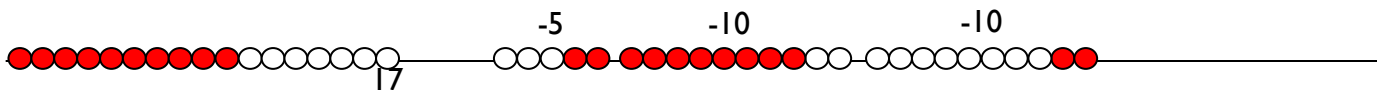
$37 + 15 =$  (shown using number sentences)

$$37 + 10 = 47$$

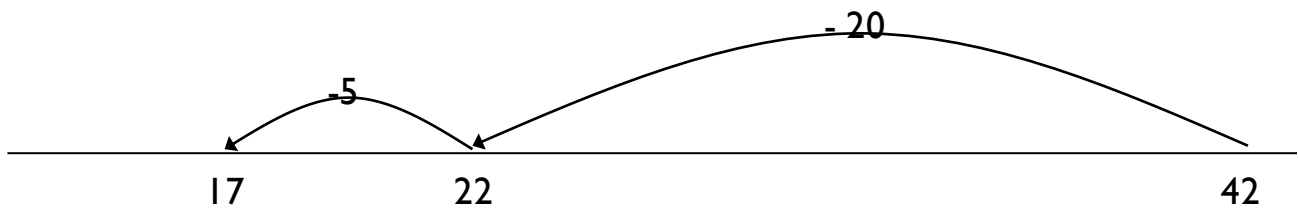
$$47 + 5 = 52$$

## Subtraction

$42 - 25 = 17$  (shown using a beadstring)



$42 - 25 = 17$  (shown using a numberline)



$42 - 25 = 17$  (shown using number sentences)

$$42 - 20 = 22$$

$$22 - 5 = 17$$

### **Reorder numbers in a calculation**

In Y3, children need to build on their knowledge gained in Y2 and continue to reorder calculations to make them more efficient. (See Y2 for more information).

*Examples of calculations:*

$23 + 54$	$54 + 23$
$12 + 19 + 12$	$12 + 12 + 19$ (using knowledge of doubles)
$6 + 8 + 4$	$6 + 4 + 8$ (using knowledge of number bonds to 10)
$70 + 50 + 30$	$70 + 30 + 50$ (using knowledge of number bonds to 100)

*Prerequisite skills:*

- Understand the place value of numbers to identify which number is the greater
- Understand that reordering works for addition but not subtraction\* (because children are not at the level when they are solving calculations such as  $16 - 3 - 6$ , when reordering would be appropriate).

### **Identify and use knowledge of number bonds within a calculation**

Number bonds to 10 and 100 can be used to make calculations more efficient when combined with other strategies such as reordering and partitioning.

*Examples of calculations:*

$42 + 38$	$42 + 30 + 8$ (recognising that 2 and 8 is a number bond to 10, so the answer will be a multiple of 10)
$60 - 28$	$60 - 20 - 8$ (utilising knowledge that $10 - 8 = 2$ , so $40 - 8 = 32$ )
$120 - 50$	$120 - 20 - 30$ (utilising knowledge of number bonds to 100, leaving an answer of 70)

*Prerequisite skills:*

- Know, or quickly derive, number bonds to 10 and 100
- Identify number bonds within other numbers, e.g. identifying  $7 + 3$  within the calculation  $57 + 33$
- Identify that when adding two two-digit numbers, that  $57 + 43 = 100$  but  $57 + 53$  does not and why

### **Find differences by counting up through the next multiple of 10 or 100**

In Y3, children need to build on their knowledge and understanding gained in Y2 to find larger differences that cross 10 and 100 boundaries. Some of these calculations are preparing children for time and money calculations throughout KS2.

*Examples of calculations:*

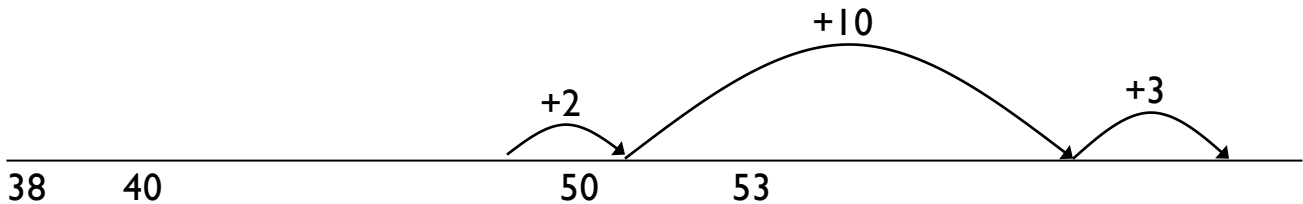
$60 - 43$	useful for time calculations, e.g. a journey time from 2:43 until 3:00
$53 - 38$	efficient because the numbers are close to each other
$104 - 95$	efficient because the numbers are close to each other
$200 - 86$	useful for money calculations, e.g. change from £2 when spending 86p

*Prerequisite skills:*

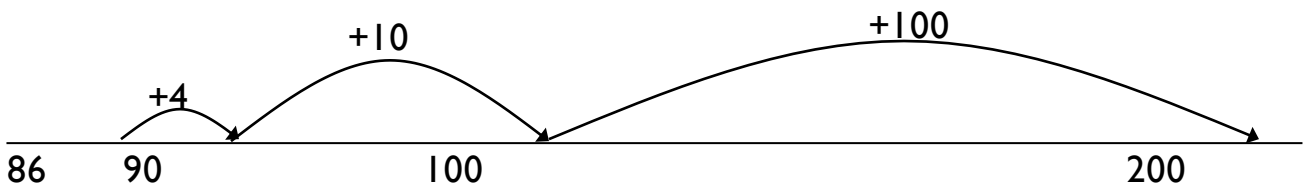
- Understand the place value of numbers to identify which number is the greater or lesser
- Establish whether numbers are close together or near to multiples of 10 or 100
- Place numbers appropriately on an unmarked numberline
- Count forwards and backwards in ones and tens

Children could use empty numberlines to record the calculation.

$$53 - 38 = 15$$



$$200 - 86 = 114$$



**Bridge through 10 when adding or subtracting a single digit number (partitioning, e.g.  $58 + 5 = 58 + 2 + 3$  or  $76 - 8 = 76 - 6 - 2$ )**

In Y3, children need to consolidate their knowledge and understanding gained in Y2. (See Y2 for more information).

*Examples of calculations*

$35 + 7$	as $35 + 5 + 2$
$97 + 6$	as $97 + 3 + 3$
$178 + 5$	as $178 + 2 + 3$
$42 - 7$	as $42 - 2 - 5$
$204 - 6$	as $204 - 4 - 2$
$371 - 5$	as $371 - 1 - 4$

*Prerequisite skills:*

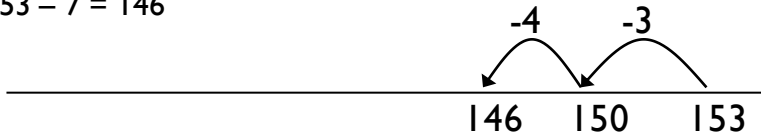
- Partition numbers in different ways, e.g. 5 as  $2 + 3$  to enable  $58 + 5$  as  $58 + 2 + 3$
- Know, or quickly derive, number bonds to 10

Children could use empty number lines to record the calculation.

$198 + 6 = 204$



$153 - 7 = 146$



The bridging strategy can then be linked with the partitioning strategy for efficient addition and subtraction of two two-digit numbers.

**Add or subtract 9, 19, 29 etc by rounding and compensating**

In Y3, children need to build on their knowledge and understanding gained in Y2 (See Y2 for more information) to add and subtract one less than a multiple of 10 up to 89 to two and three-digit numbers.

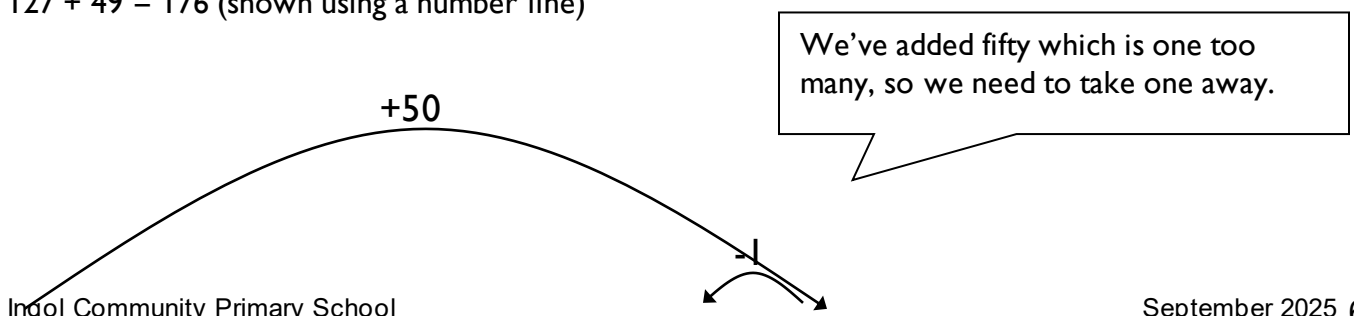
*Examples of calculations*

- 34 + 29 as 34 + 30 - 1
- 127 + 49 as 127 + 50 - 1
- 96 - 39 as 96 - 40 + 1
- 273 - 59 as 273 - 60 + 1

*Prerequisite skills:*

- Identify the difference between the number being added and subtracted and the multiple of 10
- Understand that the adjustment needs to be the opposite of the operation carried out

$127 + 49 = 176$  (shown using a number line)



---

127

176 177

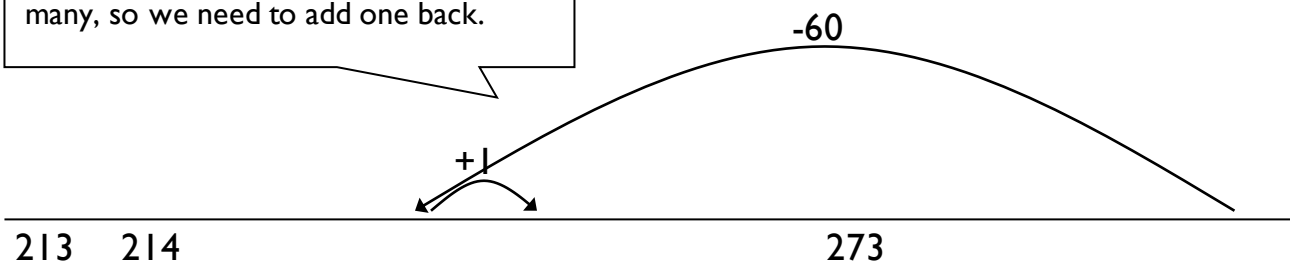
$127 + 49 = 176$  (shown using number sentences)

$127 + 50 = 177$

$177 - 1 = 176$

$273 - 59 = 214$  (shown using a number line)

We've subtracted sixty which is one too many, so we need to add one back.



$273 - 59 = 214$  (shown using number sentences)

$273 - 60 = 213$

$213 + 1 = 214$

## **Y4**

### **End of Year Objective:**

**Add and subtract numbers mentally, including: a three-digit number to or from a three-digit multiple of tens; two three-digit numbers (where there is no carrying or exchange involved)**

### **Rapid Recall:**

Children should be able to:

- recall and use addition and subtraction facts for 100
- recall and use addition and subtraction facts for multiples of 100 that total 1000
- derive and use addition and subtraction facts for 1 and 10 (with decimal numbers to one decimal place)

### **Mental Strategies**

**Partition and combine multiples of hundreds, tens and ones**

Partitioning numbers is a core strategy for adding and subtracting pairs of numbers. Children can either partition both of the numbers in the calculation, or keep the first number the same and just partition the second. (See Y2 and Y3 for more information).

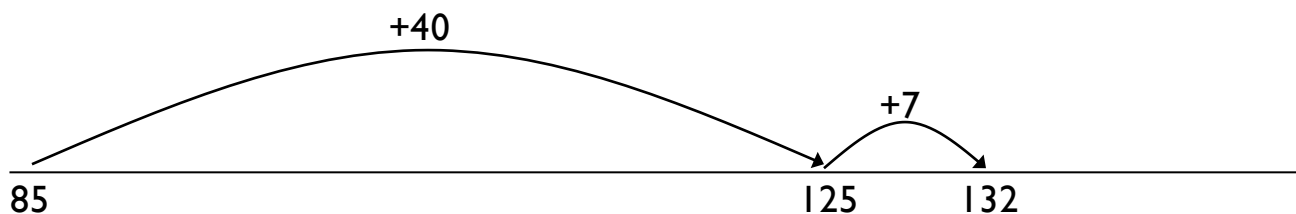
Examples of calculations:

320+150	320 add 100 and 50 = 320 add 100 add 50
243+230	243 add 200 and 30 = 243 add 200 add 30
460-140	460 take away 100 and 40 = 460 take away 100 take away 40
562 -320	562 take away 300 and 20 = 562 take away 300 take away 20
234+125	234 add 100 and 20 and 5 = 234 add 100 add 20 add 5 (crossing no boundaries)
765-241	765 take away 200 and 40 and 1 = 765 take away 200 take away 40 take away 1 (crossing no boundaries)
85 + 47	85 add 40 and 7 = 84 add 40 add 7 (crossing hundreds and tens boundaries)
122 – 35	122 take away 30 and 5 = 122 take away 30 take away 5 (crossing hundreds and tens boundaries)

Prerequisite skills:

- Count forwards and backwards in ones, tens and hundreds from any one-, two- or three-digit number
- Understand place value and understand which digit changes if one, ten or hundred is added or subtracted
- Partition numbers into hundreds, tens and ones

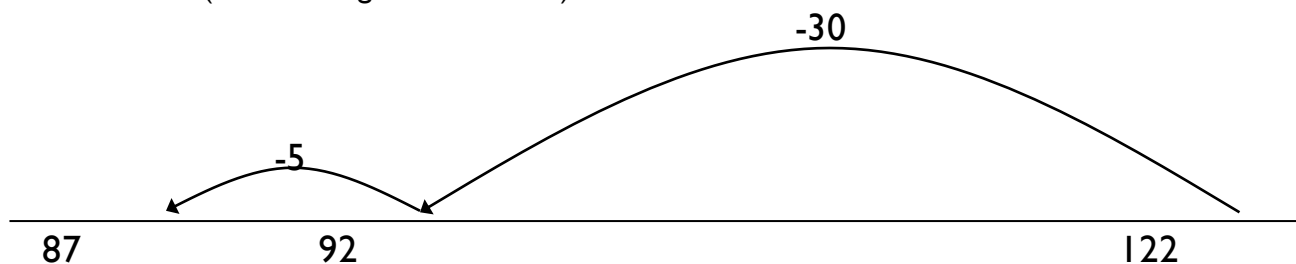
85 + 47 = 132 (shown using a number line)



85 + 47 = 132

85 + 40 = 125  
125 + 7 = 132

122 – 35 = 87 (shown using a number line)



$$122 - 35 = 87 \text{ (shown using number sentences)}$$

$$122 - 30 = 92$$

$$92 - 5 = 87$$

### **Reorder numbers in a calculation**

In Y4, children need to build on their knowledge gained in Y3 and continue to reorder calculations to make them more efficient. They should now be solving calculations involving subtraction such as  $16 - 3 - 6$ , when reordering would be appropriate.

*Examples of calculations:*

$$7 + 12 + 3 + 5$$

$$7 + 3 + 12 + 5$$

$$18 + 6 - 8$$

$$18 - 8 + 6$$

$$27 + 75$$

$$75 + 27 \text{ (thinking of 27 as } 25 + 2)$$

*Prerequisite skills:*

- Understand the place value of numbers to identify which number is the greater
- Understand that reordering works for addition but not subtraction

### **Identify and use knowledge of number bonds within a calculation and identify related facts, e.g. $150 + 270$ from $15 + 27$**

Children should use their knowledge of the number system to help them use related facts to calculate, e.g. 15 is ten times bigger than 150, 270 is ten times bigger than 27, so the answer to  $150 + 270$  will be ten times bigger than  $15 + 27$ .

*Examples of calculations:*

$$120 + 80$$

using knowledge of  $12 + 8 = 20$

$$250 + 130$$

using knowledge of  $25 + 13 = 38$

$$200 - 70$$

using knowledge of  $20 - 7 = 13$

$$460 - 150$$

using knowledge of  $46 - 15 = 31$

*Prerequisite skills:*

- Know, or quickly derive, number bonds to 10, 100 or 1000
- Identify number bonds within other numbers, e.g. identifying  $7 + 3$  within the calculation  $257 + 343$
- Identify that when adding two two-digit numbers, that  $57 + 43 = 100$  but  $57 + 53$  does not and why

### **Find differences by counting up through the next multiple of 10 or 100**

In Y4, children need to build on their knowledge and understanding gained in Y3 to find larger differences that cross 10 and 100 boundaries. When deciding whether to use a mental or a written method for a calculation, **children should be encouraged to select the method which is most efficient.** e.g.

$203 - 96 =$ . It is more efficient to count up from 96 to 203 in three steps (+4, +100, +3) than to use the formal written method of:

$$\begin{array}{r} 1 \quad 9 \quad 13 \\ 203 \\ - 96 \\ \hline 107 \end{array}$$

which requires a lot of exchanging.

Examples of calculations:

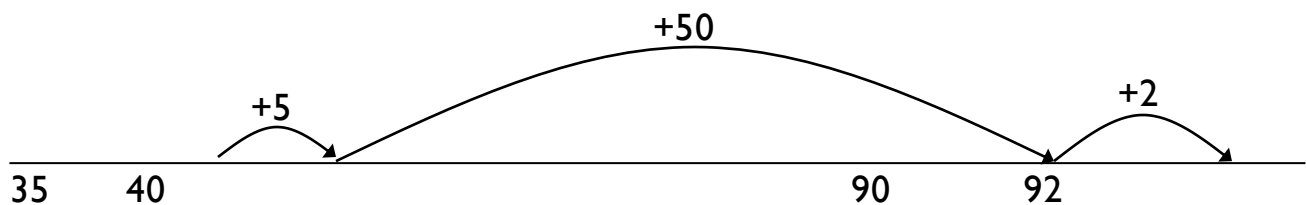
$$\begin{array}{l} 80 - 43 \\ 92 - 35 \\ 203 - 96 \\ 504 - 180 \end{array}$$

Prerequisite skills:

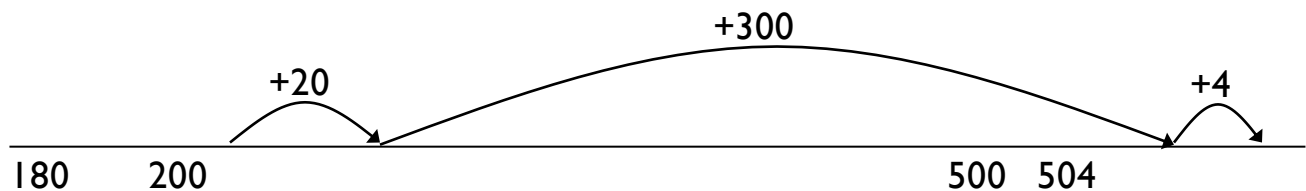
- Understand the place value of numbers to identify which number is the greater or lesser
- Establish whether numbers are close together or near to multiples of 10 or 100
- Place numbers appropriately on an unmarked number line
- Count forwards and backwards in ones and tens

Children could use empty number lines to record the calculation.

$$92 - 35 = 57$$



$$504 - 180 = 324$$



**Bridge through 10 when adding or subtracting a single digit number (partitioning, e.g.  $58 + 5 = 58 + 2 + 3$  or  $76 - 8 = 76 - 6 - 2$ )**

In Y4, children need to build on their knowledge and understanding gained in Y3. (See Y3 for more information).

Examples of calculations:

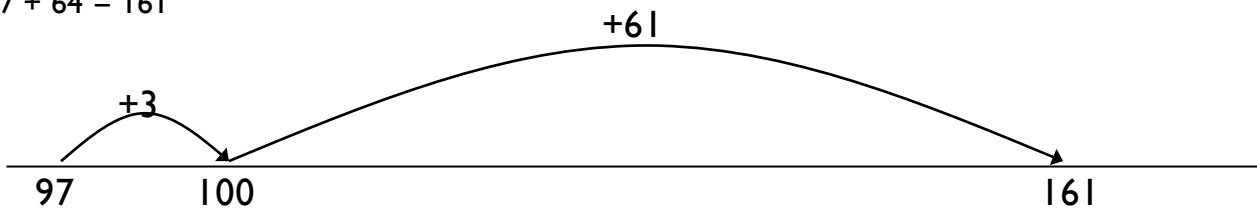
$48 + 35$  as  $48 + 2 + 33$   
 $97 + 64$  as  $97 + 3 + 61$   
 $103 - 25$  as  $103 - 3 - 22$  (using number bonds to 100)  
 $230 - 72$  as  $230 - 30 - 40 - 2$

Prerequisite skills:

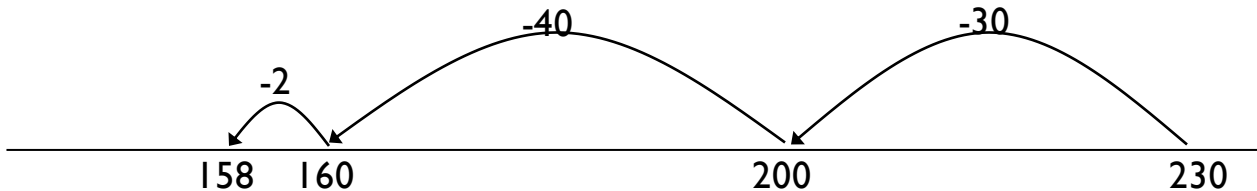
- Partition numbers in different ways, e.g. 5 as  $2 + 3$  to enable  $58 + 5$  as  $58 + 2 + 3$
- Know, or quickly derive, number bonds to 10

Children could use empty number lines to record the calculation.

$97 + 64 = 161$



$230 - 72 = 158$



**Add or subtract a multiple of 10 and adjust (for those numbers close to multiples of 10)**

In Y4, children need to build on their knowledge and understanding gained in Y3 (See Y3 for more information) to add and subtract numbers close to a multiple of 10 up to 89 to two and three-digit numbers.

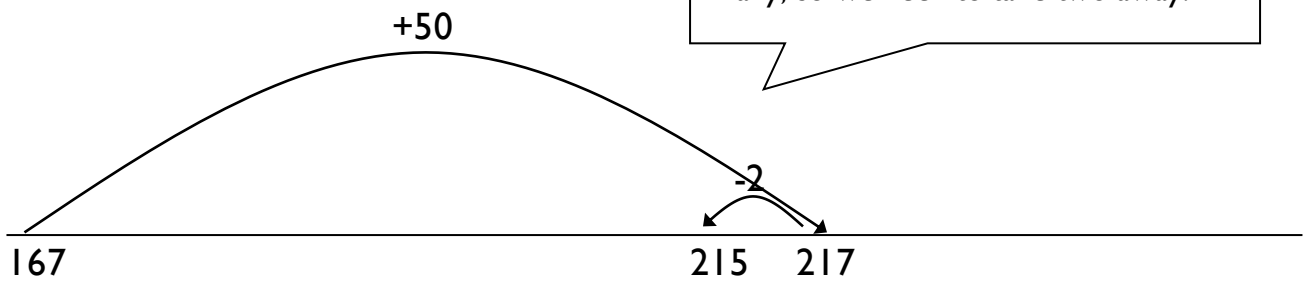
Examples of calculations:

$84 + 28$  as  $84 + 30 - 2$   
 $167 + 48$  as  $167 + 50 - 2$   
 $96 - 38$  as  $96 - 40 + 2$   
 $213 - 58$  as  $213 - 60 + 2$

Prerequisite skills:

- Identify the difference between the number being added and subtracted and the multiple of 10
- Understand that the adjustment needs to be the opposite of the operation carried out

$167 + 48 = 215$  (shown using a number line)

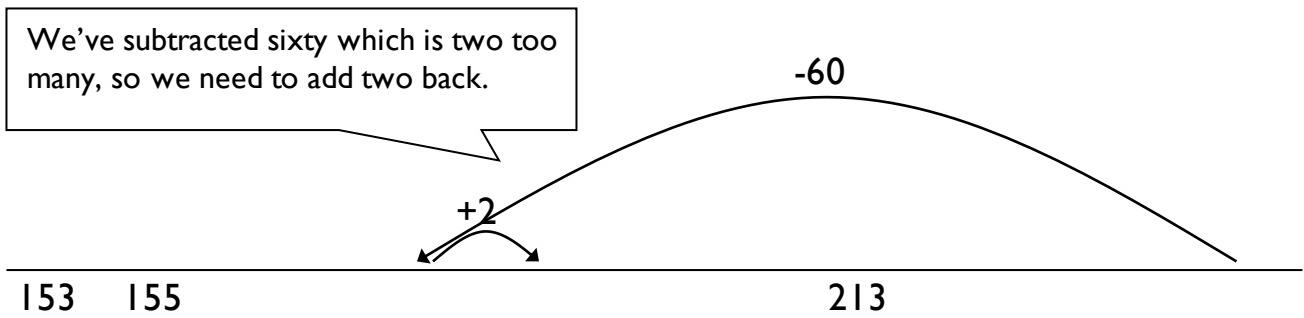


$167 + 48 = 215$  (shown using number sentences)

$$167 + 50 = 217$$

$$217 - 2 = 215$$

$213 - 58 = 155$  (shown using a number line)



$213 - 58 = 155$  (shown using number sentences)

$$213 - 60 = 153$$

$$153 + 2 = 155$$

## **Y5**

### **End of Year Objective:**

**Add and subtract numbers mentally, including: two three-digit numbers where one or both are multiples of 10 or 100; two or three-digit numbers to or from a four digit number; two four-digit numbers (where there is no carrying or exchange involved); pairs of decimals to one decimal place**

### **Rapid Recall:**

Children should be able to:

- Recall and use addition and subtraction facts for 1 and 10 (with decimal numbers to one decimal place)
- Derive and use addition and subtraction facts for 1 (with decimal numbers to two decimal places)

### **Mental Strategies**

#### ***Partition and combine multiples of thousands hundreds, tens and ones***

Partitioning numbers is a core strategy for adding and subtracting pairs of numbers. Children can either partition both of the numbers in the calculation, or keep the first number the same and just partition the second. (See Y2, Y3 and Y4 for more information).

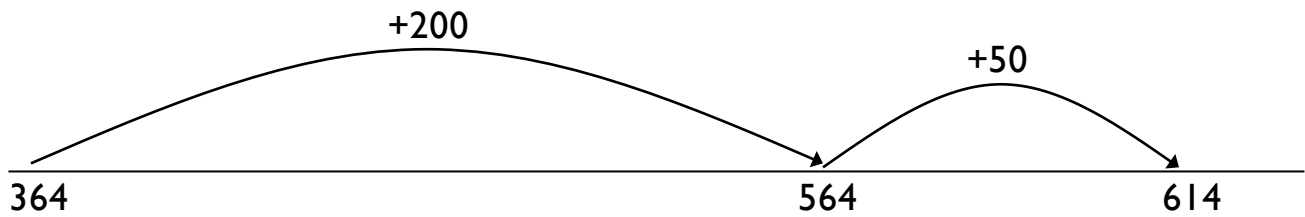
*Examples of calculations:*

4300 + 1400      4300 add 1000 and 400 = 4300 add 1000 add 400  
364 + 250      364 add 200 and 50 = 364 add 200 add 50  
3600 - 1200      3600 take away 1000 and 200 = 3600 take away 1000 take away 200  
432 - 240      432 take away 200 and 40 = 432 take away 200 take away 40  
5124 + 1352      5124 add 1000 and 300 and 50 and 2 = 5124 add 1000 add 300 add 50 add 2 (crossing no boundaries)  
7584 - 2351      7584 take away 2000 and 300 and 50 and 1 = 7584 take away 2000 take away 300 take away 50 take away 1 (crossing no boundaries)

*Prerequisite skills:*

- Count forwards and backwards in ones, tens, hundreds and thousands
- Understand place value and understand which digit changes if one, ten or hundred is added or subtracted
- Partition numbers into hundreds, tens and ones

$364 + 250 = 614$  (shown using a number line)

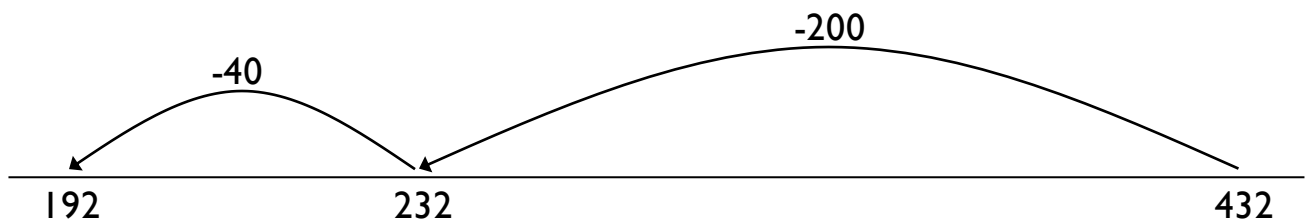


$364 + 250 = 614$  (shown using number sentences)

$$364 + 200 = 564$$

$$564 + 50 = 614$$

$432 - 240 = 192$  (shown using a number line)



$432 - 240 = 192$  (shown using number sentences)

$$432 - 200 = 232$$

$$232 - 40 = 192$$

### **Partition and combine multiples of ones and tenths**

Partitioning numbers is a core strategy for adding and subtracting pairs of numbers. Children can either partition both of the numbers in the calculation, or keep the first number the same and just partition the second. The calculations do not cross ones boundaries.

*Examples of calculations:*

$$5.4 + 3.2$$

$$5.4 \text{ add } 3 \text{ and } 0.2 = 5.4 \text{ add } 3 \text{ add } 0.2$$

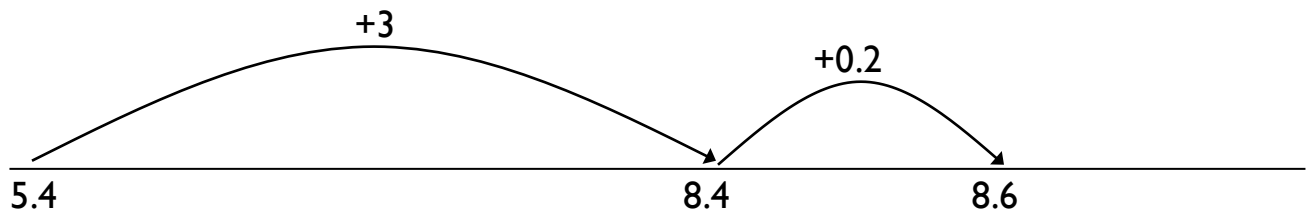
$$4.7 - 2.5$$

$$4.7 \text{ take away } 2 \text{ and } 0.5 = 4.7 \text{ take away } 2 \text{ take away } 0.5$$

Prerequisite skills:

- Count forwards and backwards in tenths and ones
- Understand place value of decimal numbers

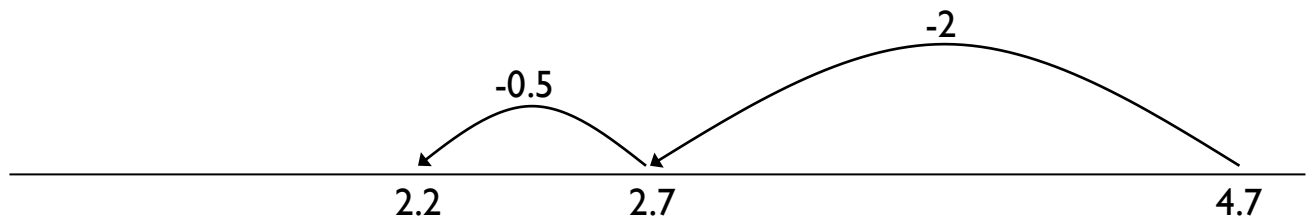
$5.4 + 3.2 = 8.6$  (shown using a number line)



$5.4 + 3.2 = 8.6$  (shown using number sentences)

$$5.4 + 3 = 8.4$$
$$8.4 + 0.2 = 8.6$$

$4.7 - 2.5 = 2.2$  (shown using a number line)



$4.7 - 2.5 = 2.2$  (shown using number sentences)

$$4.7 - 2 = 2.7$$
$$2.7 - 0.5 = 2.2$$

**Identify and use knowledge of number bonds within a calculation and identify related facts, e.g.  $1.5 + 2.7$  from  $15 + 27$**

Children should use their knowledge of the number system to help them use related facts to calculate, e.g. 1.5 is ten times smaller than 15, 2.7 is ten times smaller than 27, so the answer to  $1.5 + 2.7$  will be ten times smaller than  $15 + 27$ .

*Examples of calculations:*

$1.2 + 0.8$	using knowledge of $12 + 8 = 20$
$2.5 + 1.3$	using knowledge of $25 + 13 = 38$
$3.8 + 4.5$	using knowledge of $38 + 45 = 83$
$2 - 0.7$	using knowledge of $20 - 7 = 13$
$4.6 - 1.5$	using knowledge of $46 - 15 = 31$
$8.3 - 5.4$	using knowledge of $83 - 54 = 29$

*Prerequisite skills:*

- Know, or quickly derive, number bonds to 1, 10, 100 1000
- Identify number bonds within other numbers, e.g. identifying 7 + 3 within the calculations  $257 + 343$  or  $1.7 + 2.3$

**Find differences by counting up through the next multiple of 1, 10, 100 or 1000**

In Y5, children need to build on their knowledge and understanding gained in Y4 to find differences that cross 1, 10, 100 and 1000 boundaries. When deciding whether to use a mental or a written method for a calculation, **children should be encouraged to select the method which is most efficient.**

e.g.  $5003 - 1960 =$ . It is more efficient to count up from 1960 to 5003 in three steps (+40, +3000, +3) than to use the formal written method of:

$$\begin{array}{r} \overset{4}{5} \overset{9}{0} \overset{1}{0} 3 \\ - 1960 \\ \hline 3043 \end{array}$$

which requires a lot of exchanging.

*Examples of calculations:*

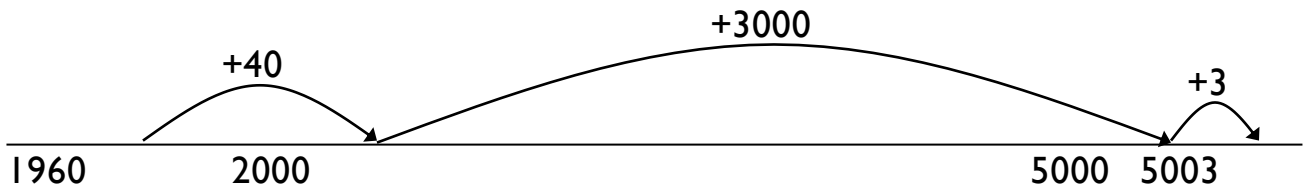
$$\begin{array}{l} 604 - 289 \\ 523 - 160 \\ 1200 - 785 \\ 5003 - 1960 \\ 7.3 - 2.8 \\ 20.1 - 6.7 \end{array}$$

*Prerequisite skills:*

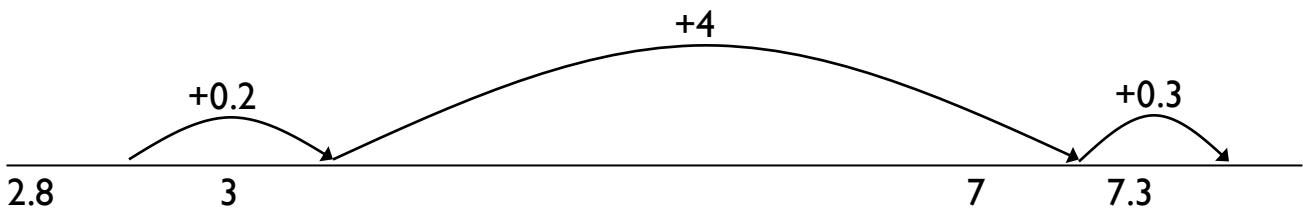
- Understand the place value of numbers to identify which number is the greater or lesser
- Establish whether numbers are close together or near to multiples of 10 or 100
- Place numbers appropriately on an unmarked number line
- Count forwards and backwards in ones and tens

Children could use empty number lines to record the calculation.

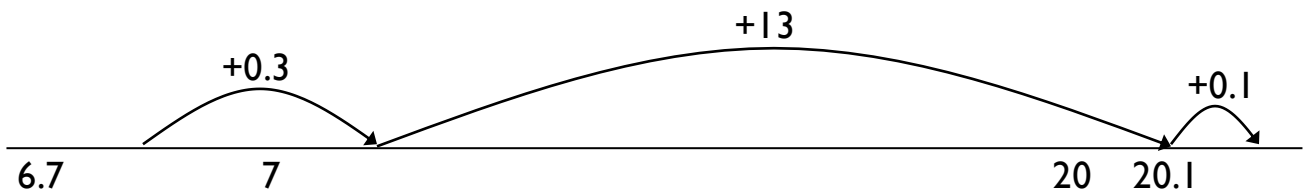
$$5003 - 1960 = 3043$$



$$7.3 - 2.8 = 4.5$$



$$20.1 - 6.7 = 13.4$$



**Bridge through 10 when adding or subtracting a single digit number (partitioning, e.g.  $58 + 5 = 58 + 2 + 3$  or  $76 - 8 = 76 - 6 - 2$ )**

In Y5, children need to build on their knowledge and understanding gained in Y4. (See Y4 for more information).

Examples of calculations:

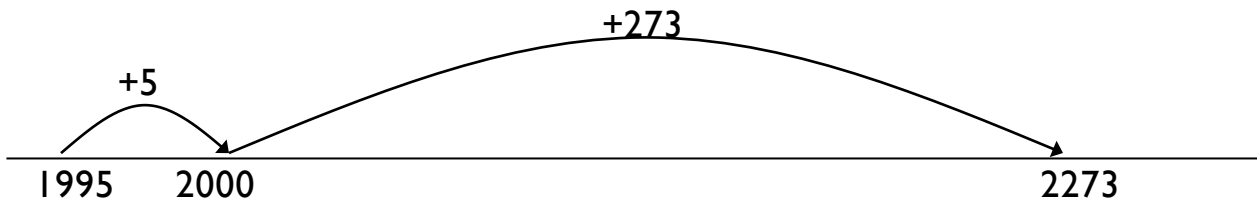
$$\begin{array}{ll} 594 + 170 & \text{as } 594 + 6 + 164 \\ 1995 + 278 & \text{as } 1995 + 5 + 273 \\ 703 - 128 & \text{as } 703 - 3 - 125 \\ 3002 - 87 & \text{as } 3002 - 2 - 85 \end{array}$$

Prerequisite skills:

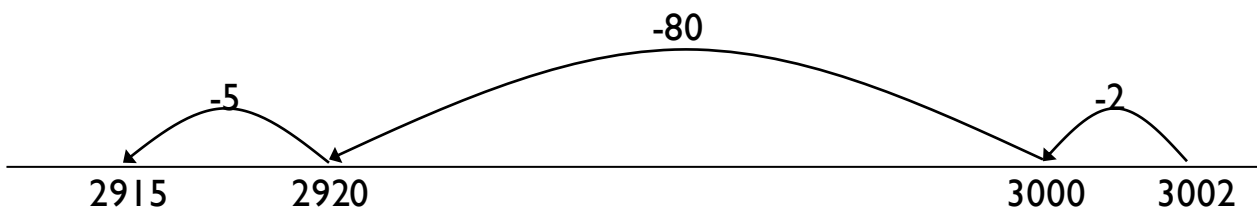
- Partition numbers in different ways, e.g. 5 as  $2 + 3$  to enable  $58 + 5$  as  $58 + 2 + 3$
- Know, or quickly derive, number bonds to 10

Children could use empty number lines to record the calculation.

$$1995 + 278 = 2273$$



$$3002 - 87 = 2915$$



**Add or subtract a multiple of 10 and adjust (for those numbers close to multiples of 10)**

In Y5, children need to build on their knowledge and understanding gained in Y4 (See Y4 for more information) to add and subtract numbers close to a multiple of 10.

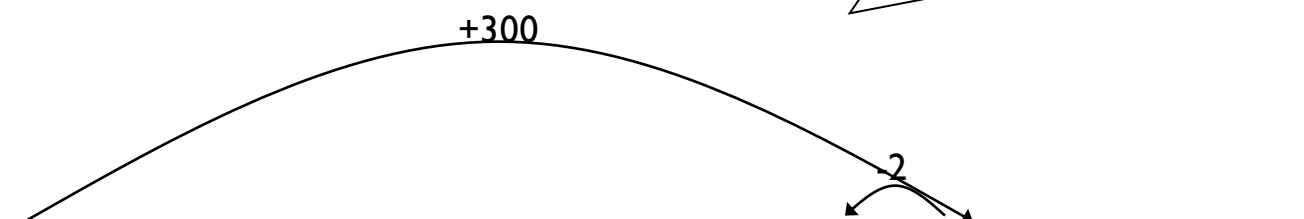
Examples of calculations:

- 257 + 68 as 257 + 70 - 2
- 325 + 298 as 325 + 300 - 2
- 764 - 88 as 764 - 90 + 2
- 876 - 397 as 876 - 400 + 3

Prerequisite skills:

- Identify the difference between the number being added and subtracted and the multiple of 10
- Understand that the adjustment needs to be the opposite of the operation carried out

$$325 + 298 = 623 \text{ (shown using a numberline)}$$



We've added three hundred which is two too many, so we need to take two away.

325

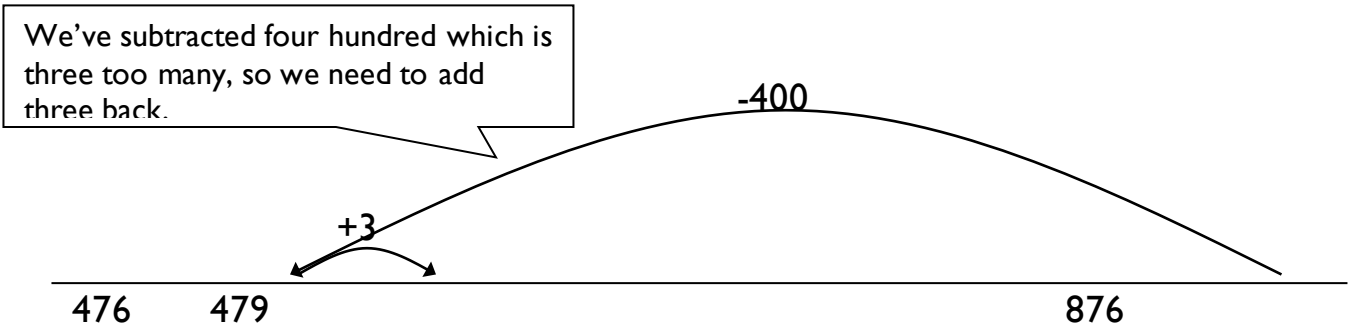
623 625

$325 + 298 = 623$  (shown using number sentences)

$325 + 300 = 625$

$625 - 2 = 623$

$876 - 397 = 479$  (shown using a number line)



$876 - 397 = 479$  (shown using number sentences)

$876 - 400 = 476$

$476 + 3 = 479$

## Y6

### **End of Year Objective:**

**Add and subtract numbers mentally, including: two three-digit numbers where one or both are multiples of 10 or 100; two or three-digit numbers to or from a four digit number; two four-digit numbers (where there is no carrying or exchange involved); pairs of decimals to one decimal place**

### **Rapid Recall:**

Children should be able to:

- Recall and use addition and subtraction facts for 1 (with decimal numbers to two decimal places)

### **Mental Strategies**

#### ***Partition and combine multiples of thousands hundreds, tens and ones***

Partitioning numbers is a core strategy for adding and subtracting pairs of numbers. Children can either partition both of the numbers in the calculation, or keep the first number the same and just partition the second. (See Y2, Y3, Y4 and Y5 for more information).

Examples of calculations:

$5800 + 2400$

5800 add 2000 and 400 = 5800 add 2000 add 400

$873 + 350$

873 add 300 and 50 = 873 add 300 add 50

$4100 - 1600$

4100 take away 1000 and 600 = 4100 take away 1000 take away 600

$2132 - 440$

2132 take away 400 and 40 = 2132 take away 400 take away 40

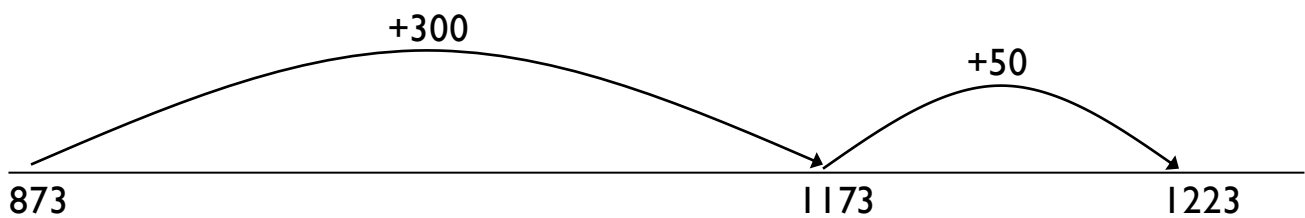
5124 + 1352 5124 add 1000 and 300 and 50 and 2 = 5124 add 1000 add 300 add 50 add 2 (crossing no boundaries)

7584 - 2351 7584 take away 2000 and 300 and 50 and 1 = 7584 take away 2000 take away 300 take away 50 take away 1 (crossing no boundaries)

Prerequisite skills:

- Count forwards and backwards in ones, tens, hundreds and thousands
- Understand place value and understand which digit changes if one, ten or hundred is added or subtracted
- Partition numbers into hundreds, tens and ones

$873 + 350 = 1223$  (shown using a number line)



$873 + 350 = 1223$  (shown using number sentences)

$873 + 300 = 1173$

$1173 + 50 = 1223$

$2132 - 440 = 1692$  (shown using a number line)



$2132 - 440 = 1692$  (shown using number sentences)

$2132 - 400 = 1732$

$1732 - 40 = 1692$

### Partition and combine multiples of ones and tenths

Partitioning numbers is a core strategy for adding and subtracting pairs of numbers. Children can either partition both of the numbers in the calculation, or keep the first number the same and just partition the second. The calculations include crossing ones boundaries. (See Year 5 for more information)

Examples of calculations:

$$8.4 + 3.8$$

$$8.4 \text{ add } 3 \text{ and } 0.8 = 8.4 \text{ add } 3 \text{ add } 0.8$$

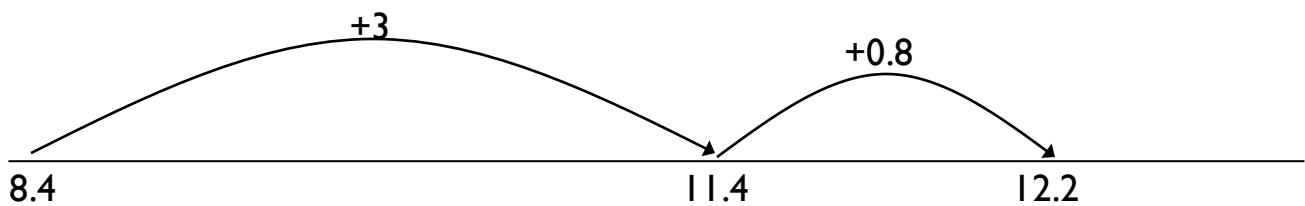
$$13.2 - 4.5$$

$$13.2 \text{ take away } 4 \text{ and } 0.5 = 13.2 \text{ take away } 4 \text{ take away } 0.5$$

Prerequisite skills:

- Count forwards and backwards in tenths and ones
- Understand place value of decimal numbers

$8.4 + 3.8 = 12.2$  (shown using a number line)

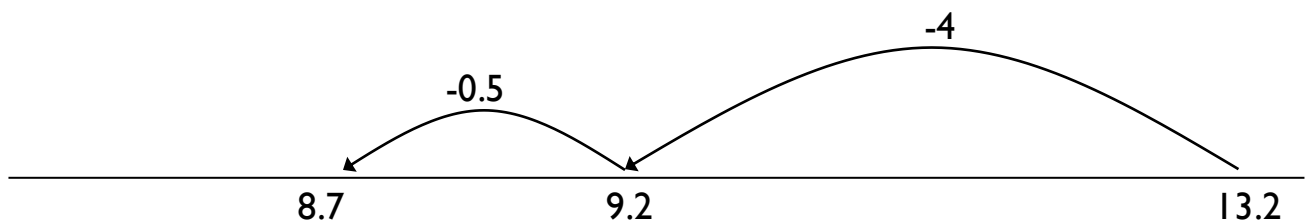


$8.4 + 3.8 = 12.2$  (shown using number sentences)

$$8.4 + 3 = 11.4$$

$$11.4 + 0.8 = 12.2$$

$13.2 - 4.5 = 8.7$  (shown using a number line)



$13.2 - 4.5 = 8.7$  (shown using number sentences)

$$13.2 - 4 = 9.2$$

$$9.2 - 0.5 = 8.7$$

**Identify and use knowledge of number bonds within a calculation and identify related facts, e.g.  $680 + 430$ ,  $6.8 + 4.3$ ,  $0.68 + 0.43$  can all be worked out using the related calculation  $68 + 43$**

In Y6, children need to build on their knowledge and understanding gained in Y5 (See Y5 for more information) Children should use their knowledge of the number system to help them use related facts to calculate, e.g. 0.68 is one hundred times smaller than 68, 0.43 is a hundred times smaller than 43, so the answer to  $0.68 + 0.43$  will be a hundred times smaller than  $68 + 43$ .

*Examples of calculations:*

$0.62 + 0.38$	using knowledge of $62 + 38 = 100$
$0.75 + 0.56$	using knowledge of $75 + 56 = 131$
$2.8 + 0.43$	using knowledge of $280 + 43 = 323$
$1 - 0.41$	using knowledge of $100 - 41 = 59$
$0.92 - 0.35$	using knowledge of $92 - 35 = 57$
$8.3 - 0.52$	using knowledge of $830 - 52 = 778$

*Prerequisite skills:*

- Know, or quickly derive, number bonds to 1, 10, 100 1000
- Identify number bonds within other numbers, e.g. identifying
- $7 + 3$  within the calculations  $257 + 343$  or  $1.7 + 2.3$

**Find differences by counting up through the next multiple of 0.1, 1, 10, 100 or 1000**

In Y6, children need to build on their knowledge and understanding gained in Y5 to find differences that cross 0.1, 10, 100 and 1000 boundaries. When deciding whether to use a mental or a written method for a calculation, children should be encouraged to select the method which is most efficient.

*Examples of calculations:*

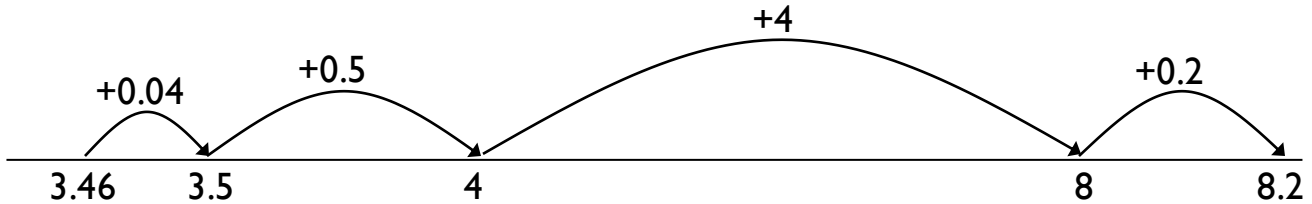
$$8.2 - 3.46$$
$$14.23 - 7.58$$

*Prerequisite skills:*

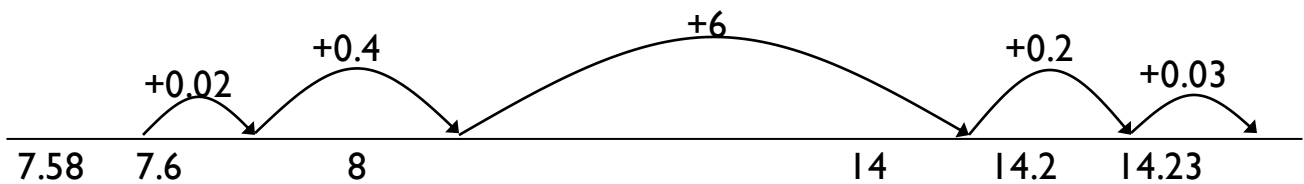
- Understand the place value of numbers to identify which number is the greater or lesser
- Establish whether numbers are close together or near to multiples of 10 or 100
- Place numbers appropriately on an unmarked number line
- Count forwards and backwards in ones and tens

Children could use empty number lines to record the calculation.

$$8.2 - 3.46 = 4.74$$



$$14.23 - 7.58 = 6.65$$



**Bridge through 10 when adding or subtracting a single digit number (partitioning, e.g.  $58 + 5 = 58 + 2 + 3$  or  $76 - 8 = 76 - 6 - 2$ )**

In Y6, children bridge using decimals to one place. To do this, it is essential that children can partition decimal numbers in different ways, e.g. 2.5 into 2 and 0.5, 2.5 into 1 and 1.5, 2.5 into 2.1 and 0.4, etc.

Examples of calculations:

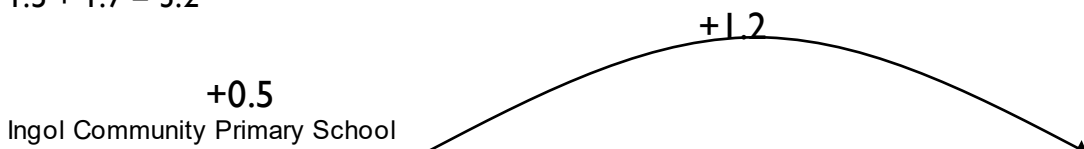
$$\begin{array}{ll} 1.5 + 1.7 & \text{as } 1.5 + 0.5 + 1.2 \\ 0.7 + 0.56 & \text{as } 0.7 + 0.3 + 0.26 \\ 8.3 - 2.7 & \text{as } 8.3 - 2.3 - 0.4 \end{array}$$

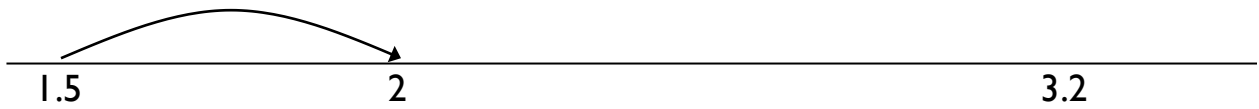
Prerequisite skills:

- Partition numbers in different ways, e.g. 5 as 2 + 3 to enable  $58 + 5$  as  $58 + 2 + 3$
- Know, or quickly derive, number bonds to 10

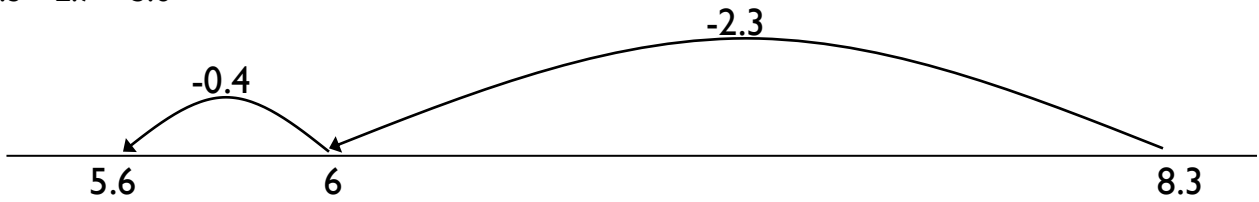
Children could use empty number lines to record the calculation.

$$1.5 + 1.7 = 3.2$$





$$8.3 - 2.7 = 5.6$$



**Add or subtract a multiple of 10 and adjust (for those numbers close to multiples of 10)**

In Y6, children adjust calculations using decimals to one place.

Examples of calculations:

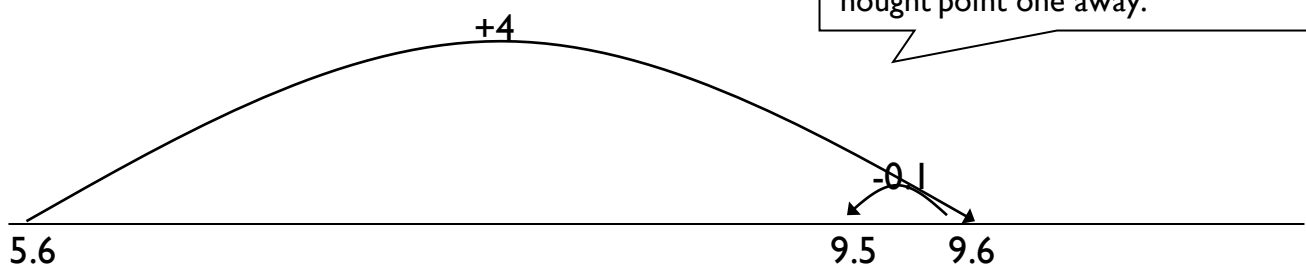
$$5.6 + 3.9 \quad - \text{ as } 5.6 + 4 - 0.1$$

$$7.5 - 4.8 \quad - \text{ as } 7.5 - 5 + 0.2$$

Prerequisite skills:

- Identify the difference between the number being added and subtracted and the multiple of 10
- Understand that the adjustment needs to be the opposite of the operation carried out

$$5.6 + 3.9 = 9.5 \text{ (shown using a number line)}$$



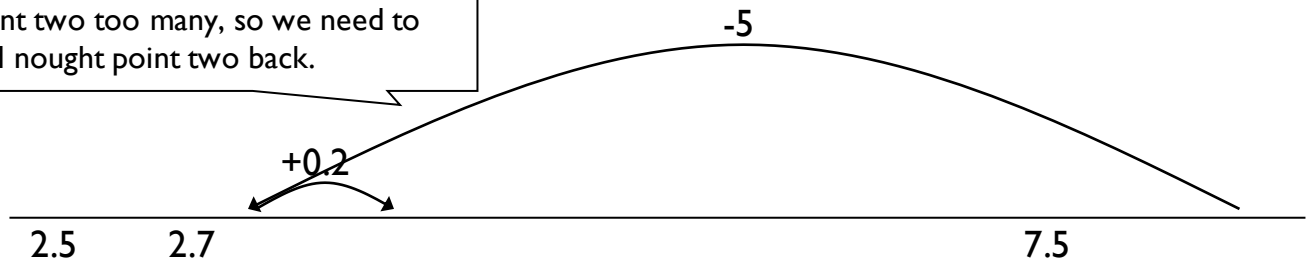
$$5.6 + 3.9 = 9.5 \text{ (shown using number sentences)}$$

$$5.6 + 4 = 9.6$$

$$9.6 - 0.1 = 9.5$$

$7.5 - 4.8 = 2.7$  (shown using a number line)

We've subtracted five which is nought point two too many, so we need to add nought point two back.



$7.5 - 4.8 = 2.7$  (shown using number sentences)

$$7.5 - 5 = 2.5$$

$$2.5 + 0.2 = 2.7$$

## **Progression Toward Mental Calculation Strategies** **(Multiplication and Division)**

The ability to calculate mentally is an essential skill, but, as with written methods of calculation, children need to be taught. **It is important to ensure that when teaching particular strategies, children have the appropriate prerequisite skills and are guided as to how and when that strategy is appropriate.**

Children should be taught and encouraged to ask themselves the following questions when faced with a calculation:

- Do I know the answer?
- Can I work it out in my head?
- Do I need to do a jotting?
- Do I need to use a written method?

When using a jotting, there is no requirement to follow a particular method of recording.

A feature of mental calculation is that a type of calculation can often be worked out in several different ways. Which method is best will depend on the numbers involved, the age of the children and the range of methods that they are confident with.

In developing a progression through mental calculation strategies for multiplication and division, it is important that children understand the relevant concepts, in that multiplication is:

- repeated addition
- scaling

and division is:

- repeated subtraction (grouping)
- related to finding a fraction of a number (sharing)

They also need to understand and work with certain principles, that:

- multiplication and division are inverses
- multiplication is commutative (because it is based on addition which is also commutative) i.e.  $3 \times 5 = 5 \times 3$  but division is not i.e.  $15 \div 3 \neq 3 \div 15$
- multiplication is associative i.e.  $2 \times (3 \times 5) = (2 \times 3) \times 5$  but division is not i.e.  $30 \div (5 \div 2) \neq (30 \div 5) \div 2$
- commutativity and associativity mean that calculations can be rearranged to make them easier to calculate, e.g.  $(3 \times 4) \times 5 = 60$  is the same as  $(5 \times 4) \times 3 = 60$

**PLEASE NOTE:** To be mathematically accurate,  $3 \times 4$  means 4 threes, or  $3 + 3 + 3 + 3$ . Read correctly it means 3 multiplied four times. The first number in the calculation is the value which is being operated on by the second:

$$3 \times 4$$

However, due to the fact that younger children often refer to the  $\times$  sign as lots of, or groups of, the calculation is then commonly represented as  $4 + 4 + 4$ . As multiplication is commutative, this is perfectly acceptable. It is a good idea to encourage children to think of any product either way round as this reduces the facts they need to remember by half.

## YR

**Early Learning Goal:**  
*Children solve problems, including doubling.*

### Rapid Recall

Children should be able to:

- Count in steps of one, forwards and backwards

In the EYFS, children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

### Solve problems involving doubling

Children may investigate items such as Lego bricks, ice cube trays and baking tins, paint boxes etc, which can show doubles, e.g. one side of the Lego brick is four, so double four is eight.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing the fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.

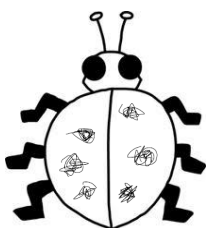


### Solve problems involving halving and sharing

Children may investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.

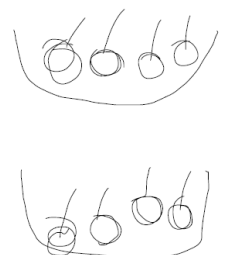


They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.

A child's jotting showing how they shared the apples at snack time between two groups.



## Y1

### **End of Year Objective:**

**Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with support.**

### **Rapid Recall**

Children should be able to:

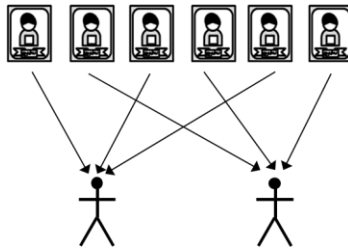
- Count in multiples of twos, fives and tens
- Recall and use doubles of all numbers to 10 and corresponding halves

### **Solve one-step problems involving multiplication**

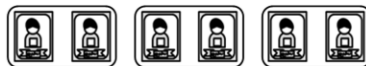
In Year One, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

### **Solve one-step problems involving division**

In Year One, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects **equally** and separate them into **equal** groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.



Children will also answer questions that involve grouping rather than sharing, such as 'Pip puts two football stickers on each page of his sticker book. How many pages does he need?' This type of problem requires the children to make equal groups from the whole amount.



## Y2

**End of Year Objective:**  
**Calculate mathematical statements for multiplication and division within the multiplication tables.**

### **Rapid Recall**

Children should be able to:

- Count in steps of 2, 3 and 5 from 0
- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables
- Derive and use doubles of simple two-digit numbers (numbers in which the ones total less than 10)
- Derive and use halves of simple two-digit even numbers (numbers in which the tens are even)
- Recognise odd and even numbers

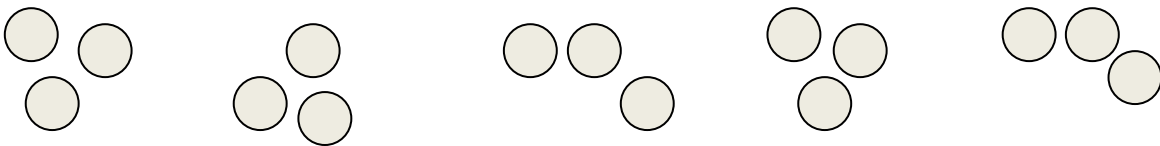
### **Mental Strategies**

Children should be able to represent a variety of multiplication and division calculations for facts not limited to two, five and ten times tables. The image and its link to the meaning of multiplication or division needs to be securely understood. As children learn to recall more multiplication and division facts, they should make a choice about the calculations they need to represent to find the answer, and those they can recall.

### **Calculate mathematical statements for multiplication**

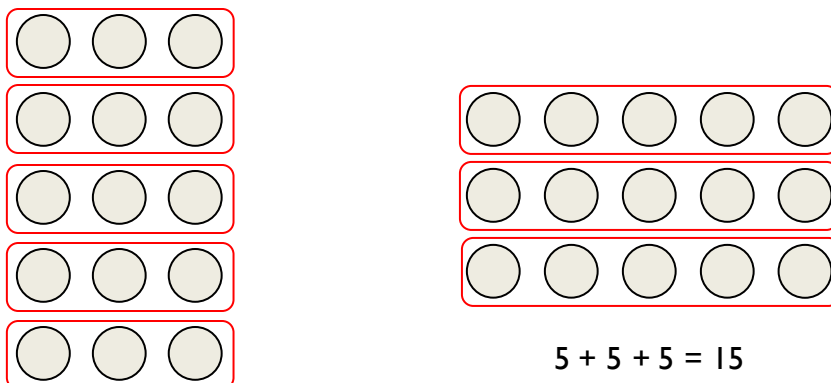
#### **Repeated addition using practical equipment**

Children should initially show multiplication as repeated addition in a random arrangement, e.g.  $5 \times 3$  can be shown as five groups of three with counters.



#### **Repeated addition using arrays**

Practical equipment can be used to represent  $5 \times 3$  as an array in two forms (as it has commutativity).



### Repeated addition using jottings

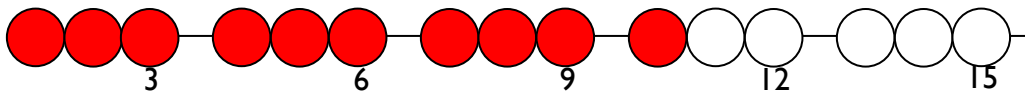
Children can develop their recordings of multiplication to using jottings.



### Repeated addition using a bead bar / bead string

Children can use bead strings or bead bars to help them calculate mathematical statements for multiplication.

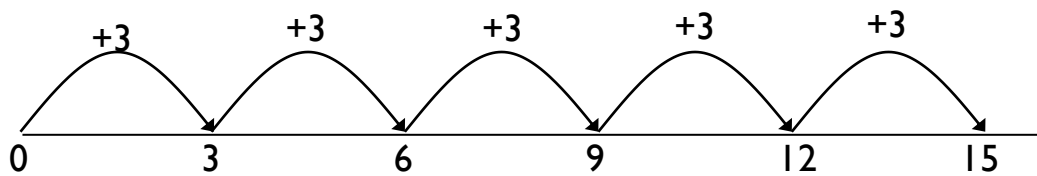
$$5 \times 3 = 5 \text{ groups of } 3$$
$$5 \times 3 = 15$$



### Repeated addition using a number line

Once children are familiar with the idea of repeated addition using a bead bar or bead string, they can begin to use a number line to show the counting steps.

$$5 \times 3 = 5 \text{ groups of } 3$$
$$5 \times 3 = 15$$



### Examples of calculations

- $5 \times 4$  using commutativity to identify a recalled fact
- $3 \times 10$  recalled fact
- $7 \times 3$  using a representation
- $2 \times 9$  using knowledge that multiplying by 2 is doubling
- $6 \times 6$  using a representation

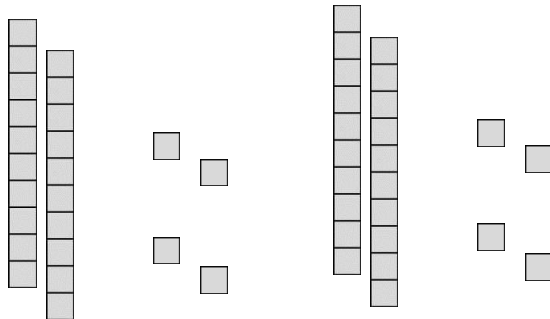
### Prerequisite skills:

- Count forwards and backwards in ones
- Identifying equal groups

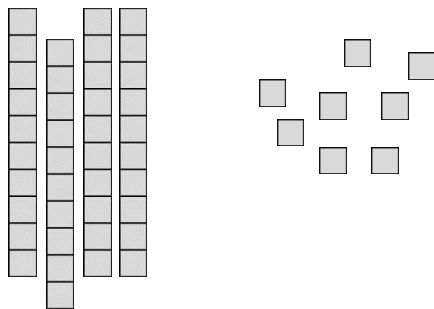
### ***Doubling using partitioning and base 10 equipment***

When deriving and using doubles of simple two-digit numbers where the ones total is less than 10, children should be taught to represent the number, and then repeat so they have two groups that are **equal in value**, developing their understanding that doubling is the same as multiplying by 2

e.g. double 24.



They should then put the tens together and the ones together:



Finally recombining the tens and ones to find the double, e.g. double 24 is 48.

### ***Doubling using partitioning and jottings***

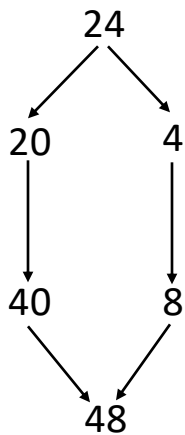
Children can then develop their use of jottings to represent the number, e.g. double 24, representing the number twice:



Finally, they can count in tens, and then ones to recombine the number, e.g. 10, 20, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48.

### ***Doubling using partitioning***

e.g. double 24



The diagram above illustrates the way children should be thinking about doubling using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

### *Examples of calculations*

Double 43

Double 31

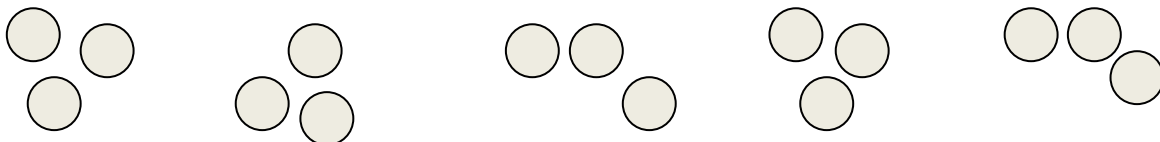
### *Prerequisite skills:*

- Count forwards and backwards in ones and tens
- Represent a number using base ten equipment
- Partition a two digit number into tens and ones
- Recombine a multiple of ten and a multiple of one

### **Calculate mathematical statements for division**

#### ***Repeated subtraction using practical equipment***

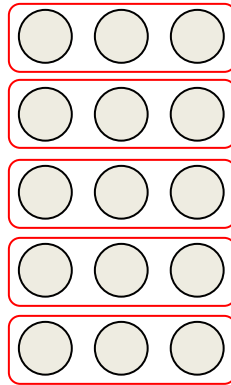
Children should initially show division as repeated subtraction in a random arrangement, e.g.  $15 \div 3$  can be shown as fifteen divided into groups of three with counters.



$15 \div 3 = 5$  groups

**Repeated subtraction using arrays**

Arrays can be used to help children to identify the groups of the divisor in a division calculation, e.g. the groups of 3 in the calculation  $15 \div 3$ .



$15 \div 3 = 5$  groups

**Repeated subtraction using jottings**

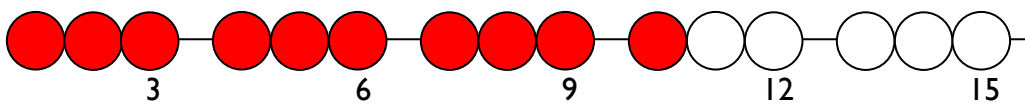
Children can develop their recordings of division to using jottings, e.g.  $15 \div 3$ ,



**Repeated subtraction using bead bar / bead string**

Children can use bead strings or bead bars to help them calculate mathematical statements for division.

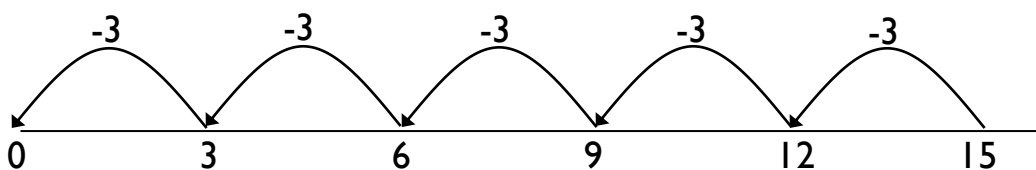
$15 \div 3 = 5$  groups



**Repeated subtraction using number line**

Once children are familiar with the idea of repeated subtraction using a bead bar or bead string, they can begin to use a number line to show the counting steps.

$15 \div 3 = 5$  groups



*Examples of calculations*

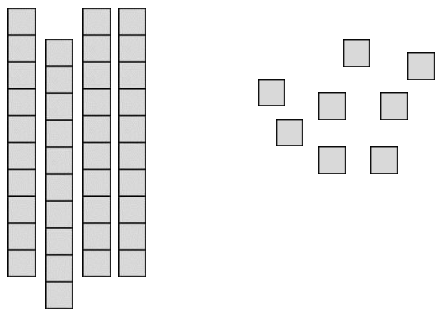
- $20 \div 5$       recalled fact
- $40 \div 10$      recalled fact
- $18 \div 3$       using a representation
- $12 \div 2$       using knowledge that dividing by 2 is halving

*Prerequisite skills:*

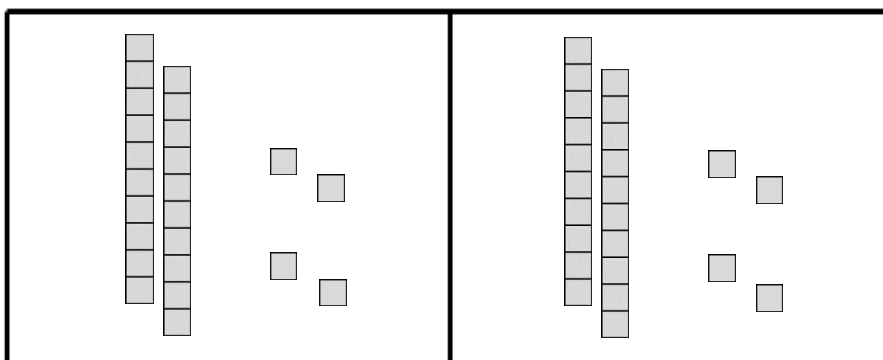
- Count forwards and backwards in ones
- Identifying equal groups

**Halving using partitioning and base 10 equipment**

When deriving and using halves of simple two-digit even numbers where the tens are even, children should first represent the number, e.g. Find half of 48



They should halve the tens and then halve the ones by sharing each of them into two equal groups, developing their understanding that halving is the same as dividing by 2. A diagram such as the one below might be useful:



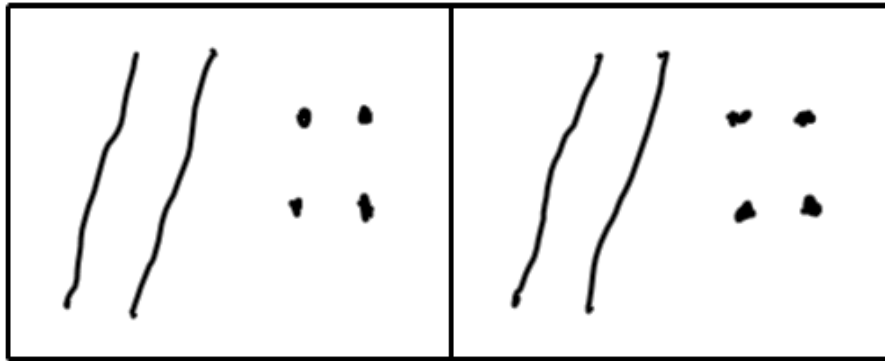
Finally, they identify what half of the whole number is, e.g. Half of 48 is 24

**Halving using partitioning and jottings**

Children can then develop their use of jottings to represent the number, e.g. Find half of 48



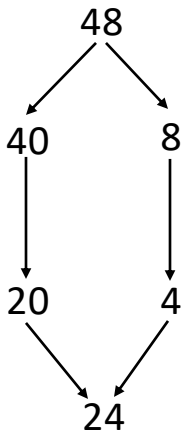
They then share each element of the number into two equal groups. A diagram such as the one below might be useful:



Finally, they identify what half of the whole number is, e.g. Half of 48 is 24.

**Halving using partitioning**

e.g. Find half of 48



The diagram above illustrates the way children should be thinking about halving using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

Find half of 64

Find half of 28

*Prerequisite skills:*

- Count forwards and backwards in ones and tens
- Represent a number using base ten equipment
- Partition a two-digit number into tens and ones
- Recombine a multiple of ten and a multiple of ones

**Y3**

**End of Year Objective:**  
*Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including two-digit numbers times one-digit numbers.*

**Rapid Recall**

Children should be able to:

- Count in multiples of 4, 8, 50 and 100
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Derive and use doubles of all numbers to 100 and corresponding halves
- Derive and use doubles of all multiples of 50 to 500

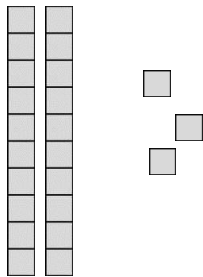
**Mental Strategies**

Children should be able to represent multiplication and division calculations, including two-digit numbers multiplied by one-digit numbers. As children learn to recall more multiplication and division facts, they should make a choice about the calculations they need to represent to find the answer, and those they can recall.

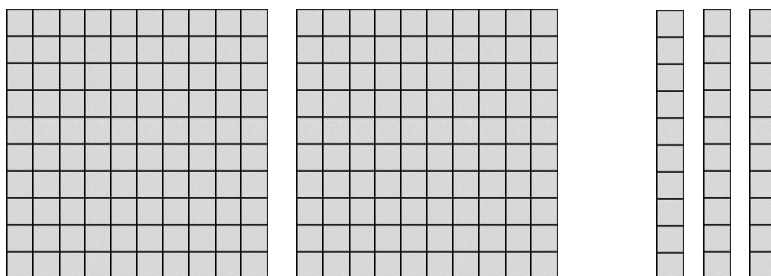
**Calculate mathematical statements for multiplication**

***Multiply a one- or two-digit number by 10 and a one-digit number by 100 using base 10 equipment***

Children should initially represent the calculation using base 10 equipment, e.g.  $23 \times 10$



All of the base 10 pieces need to be made ten times greater.



The children should then compare the two numbers in place value columns.

H	T	U
2	3	
2	3	0

They should notice that each digit has moved one place to the left, i.e. become ten times greater.

**Multiply a one- or two-digit number by 10 and a one-digit number by 100 using a place value chart**

Building on their knowledge from using the base 10 equipment, children can use transparent counters to help them develop their understanding of multiplying by 10 and 100,

e.g.  $46 \times 10$

The children represent 46 on a place value chart using transparent counters.

00000	0000	000	00	0
10000	1000	100	10	1
20000	2000	200	20	2
30000	3000	300	30	3
40000	4000	400	40	4
50000	5000	500	50	5
60000	6000	600	60	6
70000	7000	700	70	7
80000	8000	800	80	8
90000	9000	900	90	9

They then move each counter one place to the left to multiply the number by 10.

00000	0000	000	00	0
10000	1000	100	10	1
20000	2000	200	20	2
30000	3000	300	30	3
40000	4000	400	40	4
50000	5000	500	50	5
60000	6000	600	60	6
70000	7000	700	70	7
80000	8000	800	80	8
90000	9000	900	90	9

They then recombine this number to create 460.

*Examples of calculations*

$3 \times 10$

$7 \times 100$

$62 \times 10$

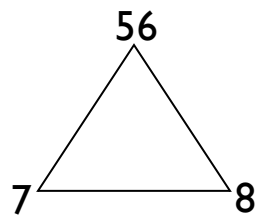
*Prerequisite skills:*

- Represent numbers up to three digits using base 10 equipment
- Partition a two digit number into tens and ones
- Recombine multiples of hundreds and tens

**Within known tables, use related facts to multiply T0 by a one-digit number**

*NB T0 represents a two digit multiple of ten*

Children should be encouraged to identify the **relationships** between numbers in multiplication calculations, e.g.  $7 \times 8 = 56$  could be represented using a multiplication trio as this model allows children to see the **relationships** between the numbers:

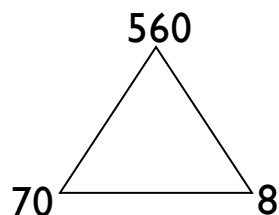


This can be used to derive the following calculations:

$7 \times 8 = 56$

$8 \times 7 = 56$

Children can then use the multiplication trio to derive related facts, e.g.  $70 \times 8 =$



Children should be able to explain that because 70 is ten times greater than 7, the answer to  $70 \times 8$  will be ten times greater than 56. They can then use their understanding of multiplying by 10 to calculate this.

*Examples of calculations*

$60 \times 3$

$50 \times 4$

$30 \times 8$

Prerequisite skills:

- Recall 2, 3, 4, 5, 8 and 10 multiplication tables
- Understand the effect of multiplying a one- or two-digit number by 10

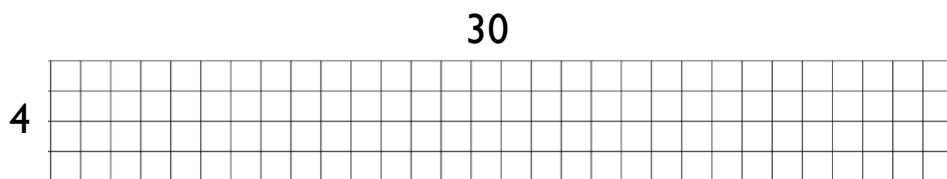
**Within known tables, use partitioning to multiply T1 by a one-digit number**

NB T1 represents a two digit number with one as the units

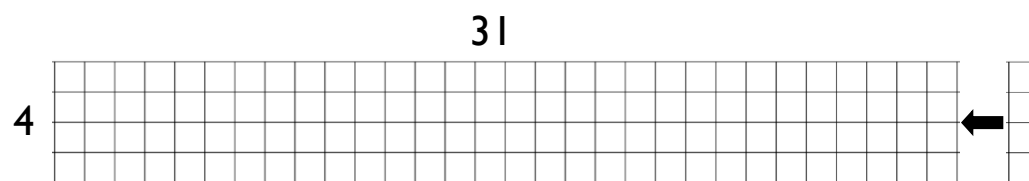
Squared paper can be used to develop children's understanding of this calculation and how it is related to multiplying T0 by a one-digit number.

e.g.  $31 \times 4$

Children make the calculation  $30 \times 4$  using squared paper.



They should be able to calculate, by using related facts, that  $30 \times 4 = 120$ . The children should now consider how to change the representation of  $30 \times 4$  into  $31 \times 4$ , i.e. by adding one extra column of four:



So  $31 \times 4 = 30 \times 4$  add  $1 \times 4$

$$31 \times 4 = 120 + 4$$

$$31 \times 4 = 124$$

*Examples of calculations*

$$51 \times 3$$

$$61 \times 4$$

$$31 \times 8$$

Prerequisite skills:

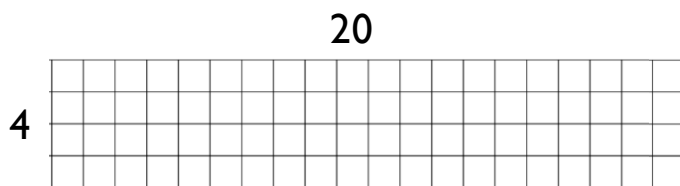
- Recall 2, 3, 4, 5, 8 and 10 multiplication tables
- Create an array to represent a multiplication calculation
- Understand the effect of multiplying a one- or two-digit number by 10
- Use related facts to multiply T0 by a one-digit number within known tables

### Use compensation to multiply 19 by a one-digit number

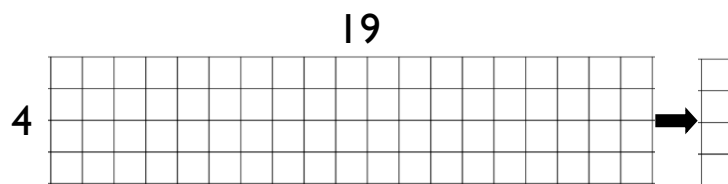
Squared paper can be used to develop children's understanding of this calculation and how it is related to multiplying twenty by a one-digit number.

e.g.  $19 \times 4$

Children make the calculation  $20 \times 4$  using squared paper.



They should be able to calculate, by using related facts, that  $20 \times 4 = 80$ . The children should now consider how to change the representation of  $20 \times 4$  into  $19 \times 4$ , i.e. by subtracting one column of four:



So  $19 \times 4 = 20 \times 4$  subtract  $1 \times 4$

$$19 \times 4 = 80 - 4$$

$$19 \times 4 = 76$$

### Examples of calculations

$$19 \times 3$$

$$19 \times 5$$

$$19 \times 8$$

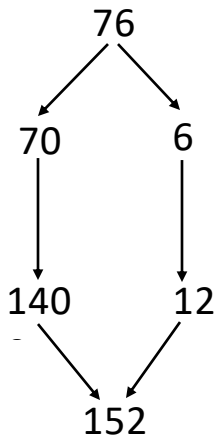
### Prerequisite skills:

- Recall 2, 3, 4, 5, 8 and 10 multiplication tables
- Create an array to represent a multiplication calculation
- Understand the effect of multiplying a one- or two-digit number by 10
- Use related facts to multiply 20 by a one-digit number within known tables

### **Use partitioning to double any two-digit number**

Children should continue to develop their understanding of doubling from Y2. They should use related facts to double two-digit multiples of 10. For example, double 7 is 14 so double 70 (ten times greater than 7) is 140 (ten times greater than 14).

e.g. double 76



The diagram above illustrates the way children should be thinking about doubling using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

### *Examples of calculations*

Double 39

Double 52

Double 85

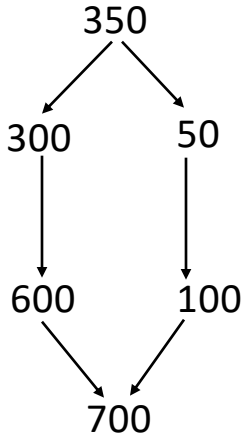
### *Prerequisite skills:*

- Count forwards in ones and tens, crossing tens and hundreds boundaries
- Partition a two-digit number into tens and ones
- Use related facts to double a two-digit multiple of 10
- Recombine a multiple of ten and a multiple of one

**Use related facts or partitioning to double any multiple of 50 to 500**

Children should use related facts to double multiples of 100. For example, double 3 is 6 so double 300 is 600.

e.g. double 350



*Examples of calculations*

Double 250

Double 450

Double 150

*Prerequisite skills:*

- Count forwards in tens and hundreds
- Partition a multiple of 50 into hundreds and tens
- Use related facts to double a multiple of 100 which is less than 500

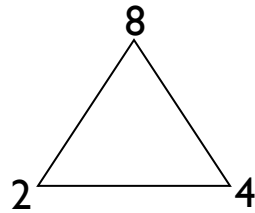
## Calculate mathematical statements for division

Children in Year 3 are learning division by chunking at this point, so calculations should fit the method being taught.

### Use related facts to divide T0 by a one-digit number

NB T0 represents a multiple of ten

Children should be encouraged to identify the **relationships** between numbers in division calculations, e.g.  $8 \div 4 = 2$  could be represented using a division trio as this model allows children to see the **relationships** between the numbers:

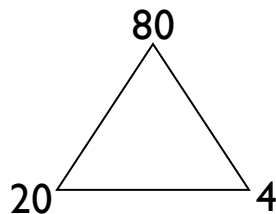


This can be used to derive the following calculations:

$$8 \div 4 = 2$$

$$8 \div 2 = 4$$

Children can then use the division trio to derive related facts, e.g.  $80 \div 4 =$



Children should be able to explain that because 80 is ten times greater than 8, the answer to  $80 \div 4$  will be ten times greater than 2.

### Examples of calculations

$$60 \div 3$$

$$80 \div 2$$

$$90 \div 3$$

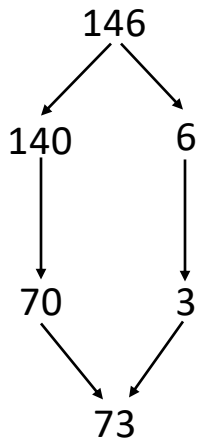
### Prerequisite skills:

- Recall 2, 3 and 4 multiplication tables
- Understand the effect of multiplying a one-digit number by 10

### Use partitioning to halve even numbers up to 200

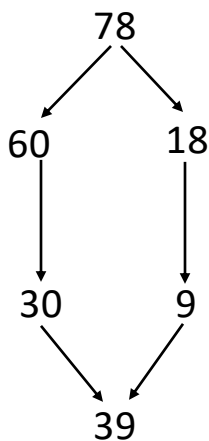
Where the tens digit is even, children can use related facts to halve the multiple of 10. For example, half of 14 is 7 so half of 140 (ten times greater than 14) is 70 (ten times greater than 7).

e.g. Find half of 146



Where the tens digit is odd, children can use partitioning in different ways to help them to calculate, recognising that partitioning the number into an even number of tens and the remainder will help them calculate more efficiently.

e.g. Find half of 78



The diagram above illustrates the way children should be thinking about halving using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

#### Examples of calculations

Find half of 162 by partitioning into 160 and 2

Find half of 94 by partitioning into 80 and 14

Find half of 136 by partitioning into 120 and 16

#### Prerequisite skills:

- Partition a two-digit and three-digit number in different ways
- Use related facts to half a multiple of 10, where the tens digit is even, up to 200
- Recombine a multiple of ten and a multiple of one

## Y4

### End of Year Objective:

Use *place value, known and derived facts to multiply and divide mentally, including:*

- *multiplying by 0 and 1*
- *dividing by 1*
- *multiplying together three numbers*

### **Rapid Recall**

Children should be able to:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Use partitioning to double or halve any number, including decimals to one decimal place
- Recognise and use factor pairs

### **Mental Strategies**

Children should be able to represent multiplication and division calculations, including two-digit multiplied by one-digit numbers. As children learn to recall more multiplication and division facts, they should make a choice about the calculations they need to represent to find the answer, and those they can recall.

### **Use place value, known and derived facts to multiply mentally**

#### ***Multiply a one- or two-digit number by 10 and 100***

Building on their understanding from Year 3, children use place value columns to multiply one or two-digit numbers by 10 and 100.

e.g.  $42 \times 100 =$

Th	H	T	U
		4	2
4	2	0	0

*Examples of calculations*

$$7 \times 10$$

$$9 \times 100$$

$$71 \times 10$$

$$63 \times 100$$

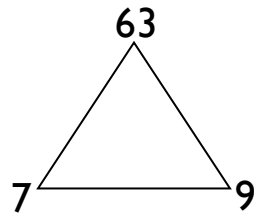
*Prerequisite skills:*

- Understand and use place value columns when representing numbers
- Understand the effect of multiplying a number by 10 or 100

### Use related facts to multiply H00 by a one-digit number

NB H00 represents a multiple of 100

Children should be encouraged to identify the **relationships** between numbers in multiplication calculations, e.g.  $7 \times 9 = 63$  could be represented using a multiplication trio as this model allows children to see the **relationships** between the numbers:

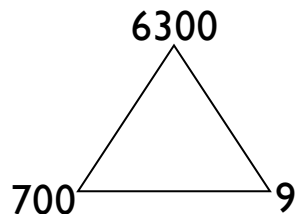


This can be used to derive the following calculations:

$$7 \times 9 = 63$$

$$9 \times 7 = 63$$

Children can then use the multiplication trio to derive related facts, e.g.  $700 \times 9 =$



Children should be able to explain that because 700 is one hundred times greater than 7, the answer to  $700 \times 9$  will be one hundred times greater than 63. They can then use their understanding of multiplying by 100 to calculate this.

### Use factor pairs to multiply H00 by a one-digit number

Calculations that involve multiplying H00 by a one-digit number can be broken down into smaller steps by using factor pairs.

e.g.  $700 \times 9 =$   
becomes  $7 \times 100 \times 9 =$  (using knowledge of factor pairs)  
which becomes  $7 \times 9 \times 100 =$  (using knowledge of commutativity/associativity)  
which becomes  $63 \times 100 = 6300$

#### Examples of calculations

$$600 \times 7$$

$$500 \times 8$$

$$900 \times 6$$

#### Prerequisite skills:

- Recall multiplication tables
- Understand the effect of multiplying a one- or two-digit number by 100
- Recognise and use factor pairs

**Use compensation to multiply T9 by a one-digit number**

NB T9 represents a two-digit number with 9 ones

Building on their understanding from Year 3 of multiplying 19 by a one-digit number using squared paper, children multiply by the nearest multiple of ten and then compensate appropriately.

e.g.  $59 \times 4 =$

$$59 \times 4 = 60 \times 4 \text{ subtract } 1 \times 4$$

$$60 \times 4 = 240$$

So  $59 \times 4 = 240 - 4$  (one group of 4 less than 240)

$$59 \times 4 = 236$$

*Examples of calculations*

$$49 \times 3$$

$$29 \times 7$$

$$89 \times 6$$

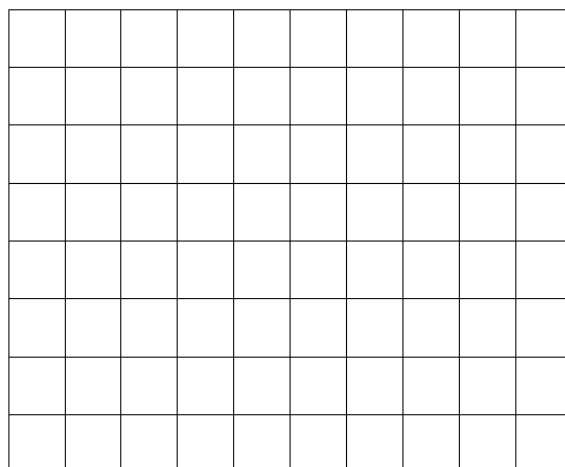
*Prerequisite skills:*

- Recall multiplication tables
- To understand how multiplying by 9 is related to multiplying by 10
- Use related facts to multiply T0 by a one-digit number

**Use related facts to multiply TU x 5 (by multiplying by 10 and halving)**

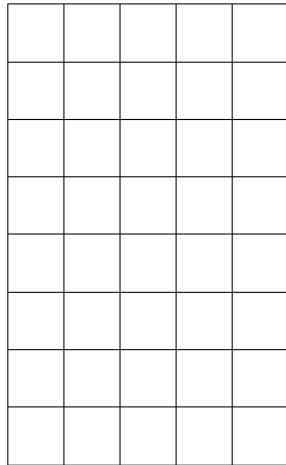
When learning multiplication tables, children should be encouraged to identify related facts.

e.g.  $8 \times 10 = 80$



This can be used to calculate:

$$8 \times 5 = 40 \quad (\text{half of } 8 \times 10) \text{ because } 5 \text{ is half of } 10$$



This strategy can then be applied to calculating  $TU \times 5$ .

e.g.  $46 \times 5 =$

$$46 \times 10 = 460$$

so

$$46 \times 5 = 230$$

*Examples of calculations*

$$28 \times 5$$

$$81 \times 5$$

$$54 \times 5$$

*Prerequisite skills:*

- Recall multiplication tables
- Understand the relationship between multiplying by 10 and multiplying by 5
- Multiply a two-digit number by 10
- Halve multiples of 10 up to three-digits

**Use related facts to multiply TU x 20 (by multiplying by 10 and doubling)**

When learning multiplication tables, children should be encouraged to identify related facts.

e.g.  $3 \times 10 = 30$


This can be used to calculate:

$3 \times 20 = 60$  (double  $3 \times 10$ ) because 20 is double 10


This strategy can then be applied to calculating TU x 20.

e.g.  $46 \times 20 =$

$46 \times 10 = 460$

so

$46 \times 20 = 920$

*Examples of calculations*

$34 \times 20$

$47 \times 20$

$68 \times 20$

*Prerequisite skills:*

- Recall multiplication tables
- Understand the relationship between multiplying by 10 and multiplying by 20
- Multiply a two-digit number by 10
- Double multiples of 10 up to three-digits

### **Use partitioning to multiply TU by a one-digit number**

Building on their understanding of the grid method of multiplication from Year 3, children in Year 4 may choose to multiply TU x U using partitioning, but without the use of the grid.

e.g.  $67 \times 4 =$

$$60 \times 4 = 240$$

$$7 \times 4 = 28$$

$$67 \times 4 = 268$$

#### *Examples of calculations*

$$57 \times 4$$

$$36 \times 7$$

$$93 \times 6$$

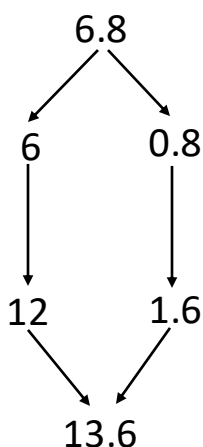
#### *Prerequisite skills:*

- Recall multiplication tables
- Partition a two-digit number into tens and ones
- Recombine a multiple of ten and a multiple of one

### **Use partitioning to double numbers including those with one decimal place**

Children should use related facts to double numbers. For example, double 7 is 14 so double 0.7 (ten times smaller than 7) is 1.4 (ten times smaller than 14).

e.g. double 6.8



The diagram above illustrates the way children should be thinking about doubling using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

Double 374

Double 4524

Double 7.6

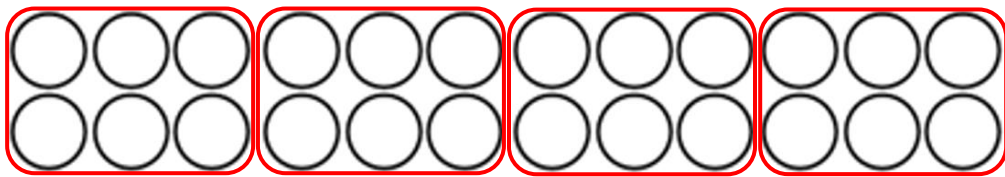
*Prerequisite skills:*

- Count forwards in tenths, ones, tens, hundreds and thousands
- Partition a number into thousands, hundreds, tens, ones and tenths
- Use related facts to double multiples of tenths, ones, tens, hundreds and thousands
- Recombine multiples of tenths, ones, tens, hundreds and thousands

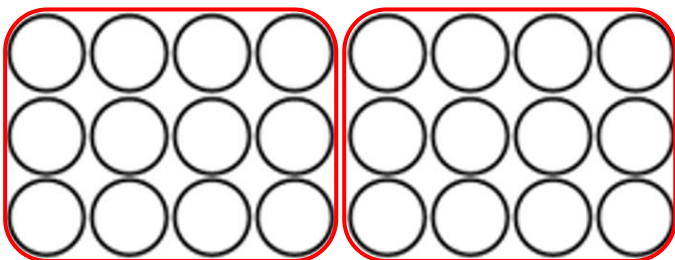
**Multiply together three numbers**

Children should be able to represent multiplying three numbers together practically, e.g. using counters

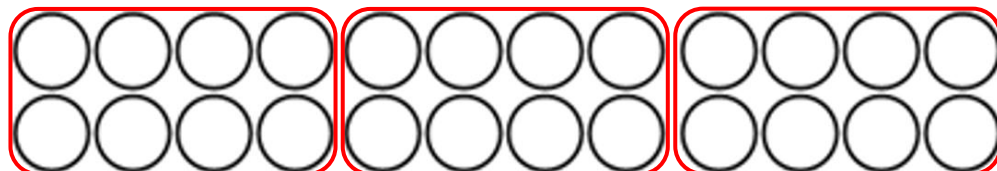
e.g.  $2 \times 3 \times 4$  could be represented as:



$2 \times 3$  four times



$3 \times 4$  two times



$2 \times 4$  three times

Once children understand how the calculation can be shown practically or pictorially, they should then be encouraged to choose an appropriate order for calculating based on the numbers involved.

e.g.  $5 \times 7 \times 4 =$

This could be calculated as:

$$\begin{array}{l} 5 \times 7 \times 4 = \\ \downarrow \\ 35 \end{array}$$

$$35 \times 4 = 140$$

$$\begin{array}{l} 5 \times 4 \times 7 = \\ \downarrow \\ 20 \end{array}$$

$$20 \times 7 = 140$$

$$\begin{array}{l} 4 \times 7 \times 5 = \\ \downarrow \\ 28 \end{array}$$

$$28 \times 5 = 140$$

Children may choose  $5 \times 4 \times 7$  as the easiest calculation because  $5 \times 4$  results in a multiple of 10.

*Examples of calculations*

$$3 \times 4 \times 6$$

$$7 \times 3 \times 9$$

$$5 \times 6 \times 8$$

*Prerequisite skills:*

- Represent a multiplication using an array
- Understand that multiplication can be done in any order
- Recall multiplication tables
- Multiply a two-digit number by a one-digit number

### ***Multiply a number by 0 or 1***

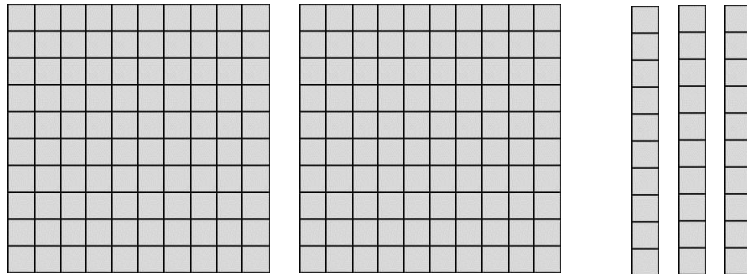
Children should realise through investigation that a calculation is not needed when multiplying by 0 or 1  
Any number multiplied by 0 will result in zero, e.g.  $76 \times 0 = 0$  because any number of empty groups does not have a value.

Any number multiplied by 1 will result in the number itself,  $356 \times 1 = 356$  because it is one group of the original amount.

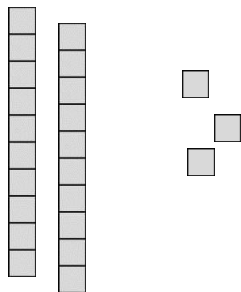
## Use place value, known and derived facts to divide mentally

### Divide a number by 10 and 100 using base 10 equipment

Children should initially represent the calculation using base 10 equipment, e.g.  $230 \div 10$



All of the base 10 pieces need to be made ten times smaller.



The children should then compare the two numbers in place value columns.

H	T	U
2	3	0
	2	3

They should notice that each digit has moved one place to the right, i.e. become ten times smaller.

### Divide a number by 10 and 100 using a place value chart

Building on their knowledge from using the base 10 equipment, children can use transparent counters to help them develop their understanding of dividing a number by 10 and 100.

e.g.  $460 \div 10$

The children represent 460 on a place value chart using transparent counters.

00000	0000	000	00	0
10000	1000	100	10	1
20000	2000	200	20	2
30000	3000	300	30	3
40000	4000	400	40	4
50000	5000	500	50	5
60000	6000	600	60	6
70000	7000	700	70	7

They then move each counter one place to the right to divide the number by 10.

00000	0000	000	00	0
10000	1000	100	10	1
20000	2000	200	20	2
30000	3000	300	30	3
40000	4000	400	40	4
50000	5000	500	50	5
60000	6000	600	60	6
70000	7000	700	70	7
80000	8000	800	80	8
90000	9000	900	90	9

*Examples of calculations*

$$120 \div 10$$

$$600 \div 100$$

$$850 \div 10$$

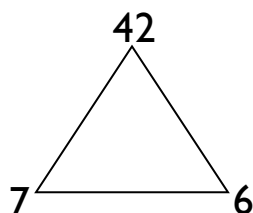
*NB There is no requirement to divide numbers where the answer is a decimal.*

*Prerequisite skills:*

- Represent numbers up to three digits using base 10 equipment
- Partition a number into hundreds and tens
- Recombine a multiple of ten and a multiple of one

**Use related facts to divide HT0 by a one-digit number**

Children should be encouraged to identify the **relationships** between numbers in division calculations, e.g.  $42 \div 7 = 6$  could be represented using a division trio:

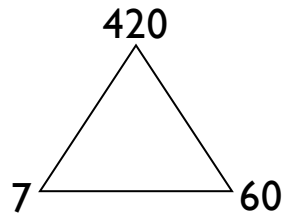


This can be used to derive the following calculations:

$$42 \div 7 = 6$$

$$42 \div 6 = 7$$

Children can then use the division trio to derive related facts, e.g.  $80 \div 4 =$



Children should be able to explain that because 420 is ten times greater than 42, the answer to  $420 \div 7$  will be ten times greater than 6.

*Examples of calculations*

$$480 \div 8$$

$$630 \div 9$$

$$300 \div 6$$

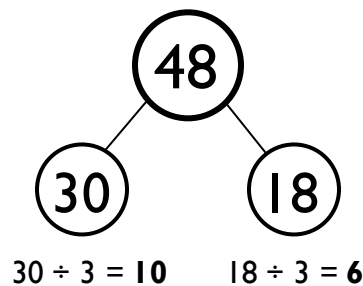
*Prerequisite skills:*

- Recall multiplication tables
- Understand the effect of multiplying a one or two-digit number by 10

**Use partitioning to divide TU by a one-digit number**

Building on their understanding of using chunking for division from Year 3, children decide how to partition a two-digit number to help them divide it by a one-digit number.

e.g.  $48 \div 3 = 16$



*Examples of calculations*

$68 \div 4$  By partitioning into 40 and 28

$95 \div 5$  By partitioning into 50 and 45

$84 \div 6$  By partitioning into 60 and 24

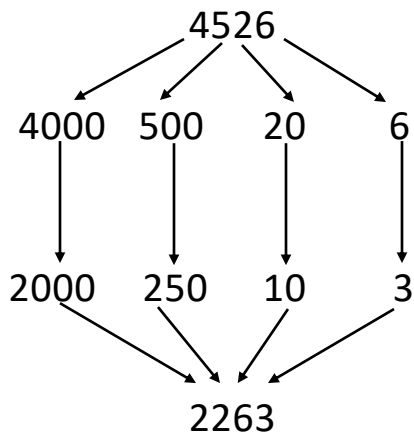
*Prerequisite skills:*

- Recall multiplication tables
- Understand division as repeated subtraction (chunking)
- Partition two-digit numbers in different ways

**Use partitioning to halve any number including to one decimal place**

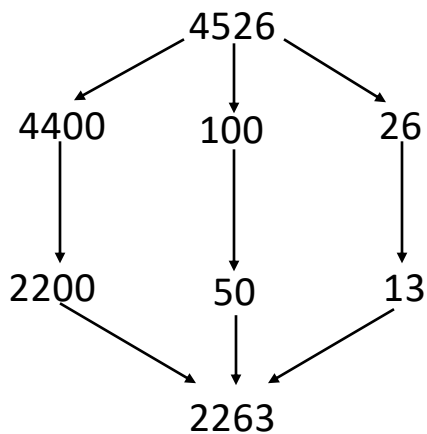
Children should be encouraged to decide the best way to partition a number to halve it.

e.g. Find half of 4526



An alternative way of partitioning would be:

Find half 4526



The diagrams above illustrate the way children should be thinking about halving using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

Find half of 468

Find half of 7602

Find half of 8.2

Find half of 3.6

*Prerequisite skills:*

- Partition numbers (including in different ways for efficiency)

- Use related facts to halve a multiple of a tenth, one, ten, hundred and thousand
- Recombine multiples of one, ten, hundred and thousand
- Recombine multiples of a tenth and one

### ***Divide a number by 1***

Children should realise, through investigation, that a calculating process is not necessary when dividing by 1

Any number that is divided by 1 will result in the number itself, e.g.  $542 \div 1 = 542$ . This is because any quantity shared into one group will result in that group having the whole quantity in it; or any number grouped into ones will result in the original number of groups.

## Y5

**End of Year Objective:**  
***Multiply and divide numbers mentally drawing upon known facts***

### **Rapid Recall**

Children should be able to:

- Recall related tables facts for multiples of 10 ( $70 \times 6 = 420$  because  $7 \times 6 = 42$ )
- Using times tables, identify related unit fractions, e.g.  $7 \times 9 = 63$  so one-ninth of 63 is 7 and one-seventh of 63 is 9
- Use partitioning to double or halve any number, including decimals to two decimal places
- Recall prime numbers up to 19
- Recall square ( $^2$ ) numbers up to  $12 \times 12$

### **Mental Strategies**

In Year 5 children build on their skills and understanding from previous year groups to multiply and divide mentally with larger numbers and numbers to two decimal places. Children should be encouraged to choose the most appropriate strategy based on the numbers involved in the calculation.

### **Multiply numbers mentally drawing upon known facts**

#### ***Multiply whole numbers and decimals to two decimal places by 10, 100 and 1000 using a place value chart***

Building on their knowledge of multiplying by 10 and 100 from Year 3 and Year 4, children can use transparent counters to help them develop their understanding of multiplying numbers to two decimal places by 10, 100 and 1000

e.g.  $3.72 \times 1000$

The children represent 3.72 on a place value chart using transparent counters.

00000	0000	000	00	0	0.0	0.00
10 000	1000	100	10	1	0.1	0.01
20 000	2000	200	20	2	0.2	0.02
30 000	3000	300	30	3	0.3	0.03
40 000	4000	400	40	4	0.4	0.04
50 000	5000	500	50	5	0.5	0.05
60 000	6000	600	60	6	0.6	0.06
70 000	7000	700	70	7	0.7	0.07
80 000	8000	800	80	8	0.8	0.08
90 000	9000	900	90	9	0.9	0.09

They then move each counter three places to the left to multiply the number by 1000 because 1000 is  $10 \times 10 \times 10$

00000	0000	000	00	0	0.0	0.00
10 000	1000	100	10	1	0.1	0.01
20 000	2000	200	20	2	0.2	0.02
30 000	3000	300	30	3	0.3	0.03
40 000	4000	400	40	4	0.4	0.04
50 000	5000	500	50	5	0.5	0.05
60 000	6000	600	60	6	0.6	0.06
70 000	7000	700	70	7	0.7	0.07
80 000	8000	800	80	8	0.8	0.08
90 000	9000	900	90	9	0.9	0.09

*Examples of calculations*

$$75.91 \times 10$$

$$5.07 \times 10$$

$$670.4 \times 100$$

$$360 \times 1000$$

$$0.76 \times 1000$$

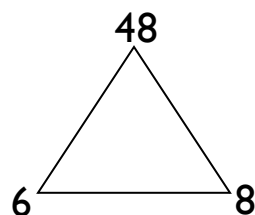
*Prerequisite skills:*

- Partition a number into thousands, hundreds, tens, ones, tenths and hundredths
- Recombine multiples of a hundred thousand, ten thousand, thousand, hundred, ten, one and tenth

**Use related facts to multiply Th000 by a one-digit number**

*NB Th000 represents a multiple of 1000*

Children should be encouraged to identify the **relationships** between numbers in multiplication calculations, e.g.  $6 \times 8 = 48$  could be represented using a multiplication trio as this model allows children to see the **relationships** between the numbers:

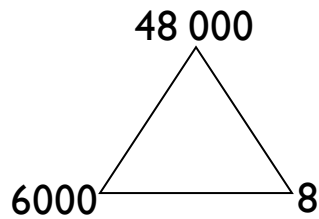


This can be used to derive the following calculations:

$$6 \times 8 = 48$$

$$8 \times 6 = 48$$

Children can then use the multiplication trio to derive related facts, e.g.  $6000 \times 8 =$



Children should be able to explain that because 6000 is one thousand times greater than 6, the answer to  $6000 \times 8$  will be one thousand times greater than 48. They can then use their understanding of multiplying by 1000 to calculate this.

*Examples of calculations*

$$3000 \times 3$$

$$7000 \times 5$$

$$8000 \times 9$$

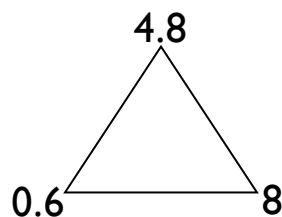
*Prerequisite skills:*

- Recall multiplication tables
- Understand the effect of multiplying a one- or two-digit number by 1000

### **Use related facts to multiply 0.t by a one-digit number**

*NB 0.t represents a multiple of a tenth*

The multiplication trio from the previous strategy can also be used to derive 0.t multiplied by a one-digit number, e.g.  $0.6 \times 8$



Children should be able to explain that because 0.6 is ten times smaller than 6, the answer to  $0.6 \times 8$  will be ten times smaller than 48. They can then use their understanding of dividing by 10 to calculate this.

*Examples of calculations*

$$0.3 \times 7$$

$$0.6 \times 9$$

$$0.5 \times 4$$

*Prerequisite skills:*

- Recall multiplication tables

- Understand the effect of dividing a one- or two-digit number by 10

**Use factor pairs to multiply T0 x T0**

Calculations that involve multiplying T0 x T0 can be broken down into smaller steps by using factor pairs.

e.g.  $60 \times 40 =$   
 becomes  $6 \times 10 \times 4 \times 10 =$  (using knowledge of factor pairs)  
 which becomes  $6 \times 4 \times 10 \times 10 =$  (using knowledge of commutativity/associativity)  
 which becomes  $24 \times 100 = 2400$

*Examples of calculations*

$30 \times 60$

$70 \times 80$

$50 \times 40$

*Prerequisite skills:*

- Recall multiplication tables
- Understand the effect of multiplying a one- or two-digit number by 100
- Recognise and use factor pairs

**Use compensation to multiply H99 by a one-digit number**

*NB H99 represents a three-digit number with 9 tens and 9 ones*

Building on their understanding from Year 4 of multiplying T9 by a one-digit number, children multiply by the nearest multiple of a hundred and then compensate appropriately.

e.g.  $699 \times 3 =$   
 $699 \times 3 = 700 \times 3$  subtract  $1 \times 3$   
 $700 \times 3 = 2100$   
 So  $699 \times 3 = 2100 - 3$  (one group of 3 less than 2100)  
 $699 \times 3 = 2097$

*Examples of calculations*

$599 \times 4$

$399 \times 7$

$699 \times 9$

*Prerequisite skills:*

- Recall multiplication tables
- Understand how multiplying by 99 is related to multiplying by 100
- Use related facts to multiply H00 by a one-digit number

- Subtract a one-digit number from a multiple of a hundred

**Use partitioning to multiply U.t by a one-digit number**

Children should be encouraged to choose the most efficient method, which may be mental, rather than simply opting for a written method.

e.g.  $3.8 \times 4 =$

$$3 \times 4 = 12$$

$$0.8 \times 4 = 3.2$$

$$3.8 \times 4 = 15.2$$

*Examples of calculations*

$$6.7 \times 4$$

$$3.2 \times 7$$

$$8.5 \times 6$$

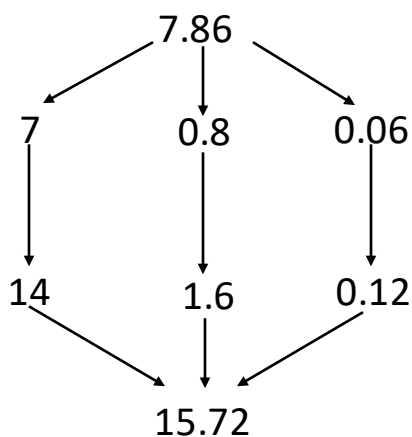
*Prerequisite skills:*

- Recall multiplication tables
- Partition U.t into ones and tenths
- Use related facts
- Add numbers with different amounts of digits

**Use partitioning to double numbers including those with two decimal places**

Children should use related facts to double numbers. For example, double 9 is 18 so double 0.09 (a hundred times smaller than 9) is 0.18 (a hundred times smaller than 18).

e.g. double 7.86



The diagram above illustrates the way children should be thinking about doubling using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

Double 56.7

Double 485.6

Double 8.59

Double 36 742

*Prerequisite skills:*

- Count forwards in hundredths, tenths, ones, tens, hundreds and thousands
- Partition a number into thousands, hundreds, tens, ones, tenths and hundredths
- Use related facts to double multiples of hundredths, tenths, ones, tens, hundreds and thousands
- Recombine multiples of hundredths, tenths, ones, tens, hundreds and thousands

**Divide numbers mentally drawing upon known facts**

***Divide whole numbers and decimals by 10, 100 and 1000 using a place value chart***

Building on their knowledge of dividing by 10 and 100 from Year 3 and Year 4, children can use transparent counters to help them develop their understanding of dividing numbers by 10, 100 and 1000. Answers should include decimals up to two decimal places.

e.g.  $35\ 600 \div 1000 =$

The children represent 35 600 on a place value chart using transparent counters.

00000	0000	000	00	0	0.0	0.00
10 000	1000	100	10	1	0.1	0.01
20 000	2000	200	20	2	0.2	0.02
30 000	3000	300	30	3	0.3	0.03
40 000	4000	400	40	4	0.4	0.04
50 000	5000	500	50	5	0.5	0.05
60 000	6000	600	60	6	0.6	0.06
70 000	7000	700	70	7	0.7	0.07
80 000	8000	800	80	8	0.8	0.08
90 000	9000	900	90	9	0.9	0.09

They then move each counter three places to the right to divide the number by 1000  
 1000 is  $10 \times 10 \times 10$  so dividing by 1000 is the same as  $\div 10 \div 10 \div 10$

00000	0000	000	00	0	0.0	0.00
10 000	1000	100	10	1	0.1	0.01
20 000	2000	200	20	2	0.2	0.02
30 000	3000	300	30	3	0.3	0.03
40 000	4000	400	40	4	0.4	0.04
50 000	5000	500	50	5	0.5	0.05
60 000	6000	600	60	6	0.6	0.06
70 000	7000	700	70	7	0.7	0.07
80 000	8000	800	80	8	0.8	0.08
90 000	9000	900	90	9	0.9	0.09

*Examples of calculations*

$874 \div 10$

$60.1 \div 10$

$7043 \div 100$

$48\,750 \div 1000$

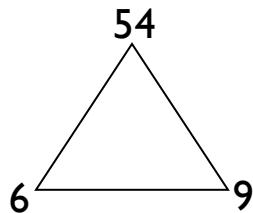
*Prerequisite skills:*

- Partition a number into hundred thousands, ten thousands, thousands, hundreds, tens, ones and tenths
- Recombine multiples of ten thousand, thousand, hundred, ten, one, tenth and hundredth

**Use related facts to divide ThH00 by a one-digit number**

*NB ThH00 represents a four-digit multiple of 100*

Children should be encouraged to identify the **relationships** between numbers in division calculations, e.g.  $54 \div 6 = 9$  could be represented using a division trio:

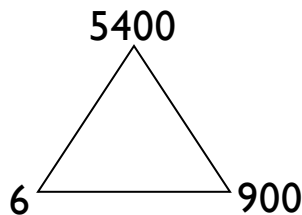


This can be used to derive the following calculations:

$$54 \div 6 = 9$$

$$54 \div 9 = 6$$

Children can then use the division trio to derive related facts, e.g.  $5400 \div 6 = 900$



Children should be able to explain that because 5400 is a hundred times greater than 54, the answer to  $5400 \div 6$  will be a hundred times greater than 9. They can then use their understanding of multiplying by 100 to calculate this.

*Examples of calculations*

$$8100 \div 9$$

$$3000 \div 6$$

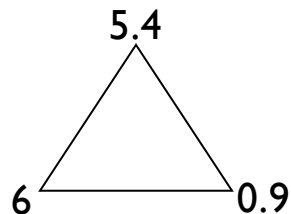
$$9600 \div 8$$

*Prerequisite skills:*

- Recall multiplication tables
- Understand the effect of multiplying a one or two-digit number by 100

### **Use related facts to divide U.t by a one-digit number**

The division trio from the previous strategy can also be used to derive U.t divided by a one-digit number, e.g.  $5.4 \div 6 =$



Children should be able to explain that because 5.4 is ten times smaller than 54, the answer to  $5.4 \div 6$  will be ten times smaller than 9. They can then use their understanding of dividing by 10 to calculate this.

#### *Examples of calculations*

$$2.1 \div 7$$

$$3.6 \div 9$$

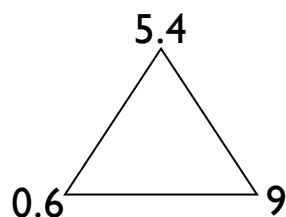
$$4.8 \div 4$$

#### *Prerequisite skills:*

- Recall multiplication tables
- Understand the effect of dividing a one- or two-digit number by 10

### **Use related facts to divide U.t by a 0.t**

The division trio from the previous strategy can also be used to derive U.t divided by 0.t, e.g.  $5.4 \div 0.6 =$



Children should be able to explain that 5.4 is ten times smaller than 54 and 0.6 is ten times smaller than 6. This means that both numbers have been scaled down by the same amount, so the relationship between the numbers stays the same. The answer to  $5.4 \div 0.6$  will therefore be 9 because there are 9 groups of 0.6 in 5.4

#### *Examples of calculations*

$$2.1 \div 0.7$$

$$3.6 \div 0.9$$

$$4.8 \div 0.4$$

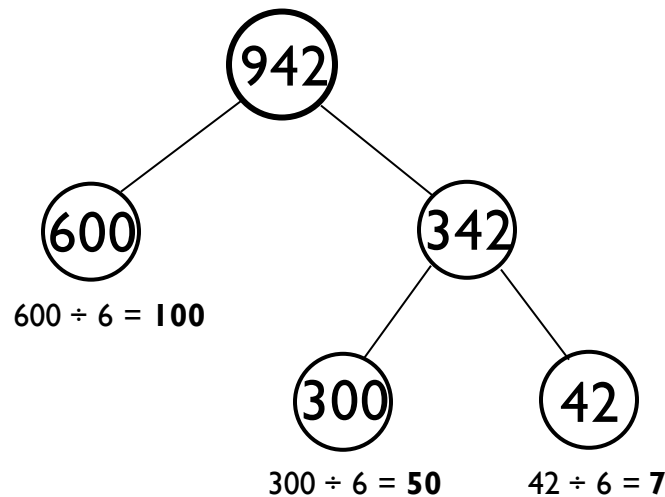
*Prerequisite skills:*

- Recall multiplication tables
- Understand division as repeated subtraction

**Use partitioning to divide HTU by a one-digit number**

Building on their understanding of using partitioning to divide TU by a one-digit number from Year 4, children decide how to partition HTU to help them divide it by a one-digit number.

e.g.  $942 \div 6 = 157$



The diagram above illustrates the way children should be thinking about dividing using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

- $756 \div 9$  By partitioning into 720 and 36
- $765 \div 5$  By partitioning into 500 and 250 and 15
- $861 \div 7$  By partitioning into 700 and 140 and 21

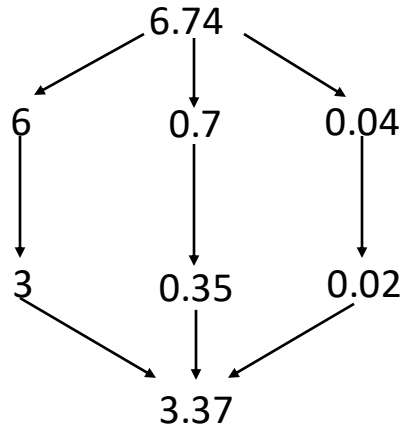
*Prerequisite skills:*

- Recall multiplication tables
- Understand division as repeated subtraction
- Partition three-digit numbers in different ways

**Use partitioning to halve any number including to two decimal places**

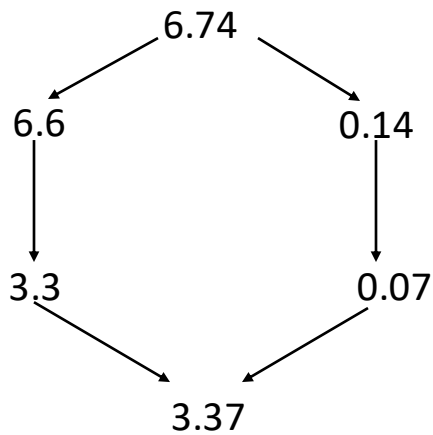
Children should be encouraged to decide the best way to partition a number to halve it.

e.g. Find half of 6.74



An alternative way of partitioning would be:

Find half of 6.74



The diagrams above illustrate the way children should be thinking about halving using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

Find half of 4.62

Find half of 18.46

Find half of 8.94

Find half of 17.92

Find half of 32 784

*Prerequisite skills:*

- Partition numbers (including in different ways for efficiency)
- Use related facts to halve a multiple of a hundredth, tenth, ten, hundred and thousand
- Recombine multiples of one, ten, hundred and thousand
- Recombine multiples of a tenth and a hundredth

## **Y6**

### **End of Year Objective:**

***Perform mental calculations, including with mixed operations and large numbers***

### **Rapid Recall**

Children should be able to:

- Recall related tables facts decimal numbers ( $0.7 \times 6 = 4.2$  because  $7 \times 6 = 42$ )
- Use partitioning to double or halve any number
- Recall prime numbers up to 100
- Recall squares of the corresponding multiples of 10 (i.e.  $40^2$  is 1600)

### **Mental Strategies**

In Year 6 children build on their skills and understanding from previous year groups to multiply and divide mentally with larger numbers and numbers to three decimal places. Children should be encouraged to choose the most appropriate strategy based on the numbers involved in the calculation.

### **Perform mental calculations – Multiplication**

#### ***Multiply whole numbers and decimals to three decimal places by 10, 100 and 1000***

Building on their knowledge of multiplying by 10, 100 and 1000 from Year 5, children use place value columns to multiply numbers to three decimal places by 10, 100 and 1000

e.g.  $43.721 \times 100 =$

Th	H	T	U • t	h	th
		4	3 • 7	2	1
4	3	7	2 • 1		

#### *Examples of calculations*

$$4562 \times 1000$$

$$9.682 \times 10$$

$$25.784 \times 100$$

#### *Prerequisite skills:*

- Understand and use place value columns when representing numbers

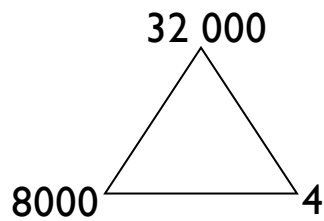
- Understand the effect of multiplying a number by 10, 100 or 1000

**Identify and use all related facts that link to tables**

Children should be encouraged to select the most appropriate strategy based on the numbers involved in the calculation.

e.g.  $8000 \times 4 =$

Using related facts in a multiplication trio could help with this calculation:



Children should be able to explain that because 8000 is a thousand times greater than 8, the answer to  $8000 \times 4$  will be a thousand times greater than 32.

e.g.  $8000 \times 40 =$

Using factor pairs could help with this calculation:

	$8000 \times 40 =$	
becomes	$8000 \times 4 \times 10 =$	(using knowledge of factor pairs)
which becomes	$32\ 000 \times 10 = 320\ 000$	

*Examples of calculations*

$7000 \times 6$

$500 \times 40$

$900 \times 300$

$3000 \times 80$

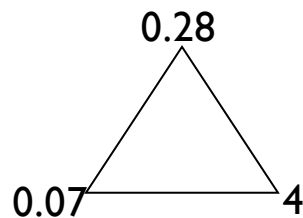
*Prerequisite skills:*

- Recall multiplication tables
- Understand the effect of multiplying a number by 10, 100 or 1000
- Recognise and use factor pairs

**Use related facts to multiply 0.0t by a one-digit number**

NB 0.0t represents a multiple of a hundredth

Children can use a multiplication trio to derive related facts, e.g.  $0.07 \times 4$



Children should be able to explain that because 0.07 is a hundred times smaller than 7, the answer to  $0.07 \times 4$  will be a hundred times smaller than 28. They can then use their understanding of dividing by 100 to calculate this.

*Examples of calculations*

$$0.03 \times 7$$

$$0.06 \times 9$$

$$0.05 \times 4$$

*Prerequisite skills:*

- Recall multiplication tables
- Understand the effect of dividing a one- or two-digit number by 100

**Use compensation to multiply U.9 and U.99 by a one-digit number**

Building on their understanding from Year 5 of multiplying H99 by a one-digit number, children multiply by the nearest whole number and then compensate appropriately.

e.g.  $6.9 \times 4 =$

$$6.9 \times 4 = 7 \times 4 \text{ subtract } 0.1 \times 4$$

$$7 \times 4 = 28$$

So  $6.9 \times 4 = 28 - 0.4$

$$6.9 \times 4 = 27.6$$

e.g.  $6.99 \times 4 =$

$$6.99 \times 4 = 7 \times 4 \text{ subtract } 0.01 \times 4$$

$$7 \times 4 = 28$$

So  $6.99 \times 4 = 28 - 0.04$

$$6.99 \times 4 = 27.96$$

*Examples of calculations*

$$5.9 \times 4$$

$$3.99 \times 7$$

$$9.99 \times 6$$

*Prerequisite skills:*

- Recall multiplication tables
- Understand how multiplying by 0.9 is related to multiplying by 1
- Understand how multiplying by 0.99 is related to multiplying by 1
- Subtract a 0.t or 0.0h from a whole number

**Use partitioning to multiply 0.th by a one-digit number**

Children should be encouraged to choose the most efficient method, which may be mental, rather than simply opting for a written method.

e.g.  $0.67 \times 4 =$

$$0.6 \times 4 = 2.4$$

$$0.07 \times 4 = 0.28$$

$$0.67 \times 4 = 2.68$$

*Examples of calculations*

$$0.76 \times 3$$

$$0.28 \times 7$$

$$0.54 \times 6$$

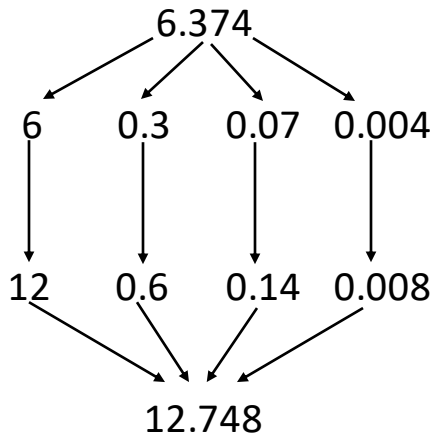
*Prerequisite skills:*

- Recall multiplication tables
- Partition 0.th into tenths and hundredths
- Use related facts
- Add numbers with different amounts of digits

**Use partitioning to double numbers including those with three decimal places**

Children should use related facts to double numbers. For example, double 9 is 18 so double 0.009 (a thousand times smaller than 9) is 0.018 (a thousand times smaller than 18).

e.g. double 6.374



The diagram above illustrates the way children should be thinking about doubling using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

Double 3.421

Double 6.705

Double 12.594

Double 54 672

Double 674 960

*Prerequisite skills:*

- Count forwards in steps of powers of 10
- Partition a number appropriately
- Use related facts to double multiples of powers of 10
- Recombine multiples of powers of 10

## Perform mental calculations – Division

### **Divide whole numbers and decimals to three decimal places by 10, 100 and 1000**

Building on their knowledge of dividing by 10, 100 and 1000 from Year 5, children use place value columns to divide numbers by 10, 100 and 1000. Answers should include decimals up to three decimal places.

e.g.  $356.7 \div 100 =$

Th	H	T	U • t	h	th
	3	5	6 • 7		
			3 • 5	6	7

#### *Examples of calculations*

$$9.83 \div 10$$

$$7.04 \div 10$$

$$860.2 \div 100$$

$$56\,789 \div 1000$$

#### *Prerequisite skills:*

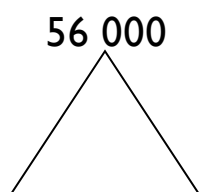
- Understand and use place value columns when representing numbers
- Understand the effect of dividing a number by 10, 100 or 1000

### **Identify and use all related facts that link to tables**

Children should be encouraged to select the most appropriate strategy based on the numbers involved in the calculation.

e.g.  $56\,000 \div 8 =$

Using related facts in a division trio could help with this calculation:



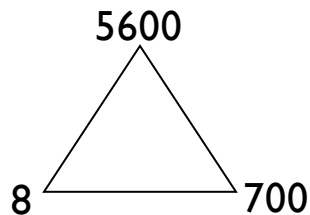
8

7000

Children should be able to explain that because 56 000 is a thousand times greater than 56, the answer to  $56\ 000 \div 8$  will be a thousand times greater than 7. They can then use their understanding of multiplying by 1000 to calculate this.

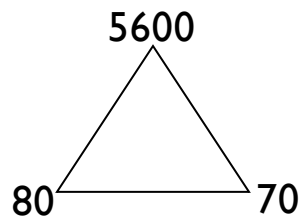
e.g.  $5600 \div 80 =$

The following division trio could be used as a starting point for this calculation:



Children should be able to explain that because 5600 is a hundred times greater than 56, the answer to  $5600 \div 8$  will be a hundred times greater than 7. They can then use their understanding of multiplying by 100 to calculate this.

The following division trio could then be derived:



Children should be able to explain that because 80 is ten times greater than 8, the answer to  $5600 \div 80$  will be ten times smaller than the answer to  $5600 \div 8$  because there will be ten times fewer groups.

#### *Examples of calculations*

$$81\ 000 \div 9$$

$$270 \div 30$$

$$3000 \div 50$$

$$9600 \div 800$$

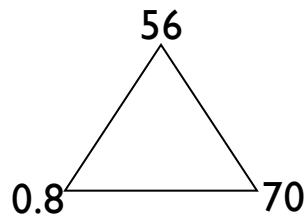
#### *Prerequisite skills:*

- Recall multiplication tables
- Understand division as repeated subtraction
- Understand the effect of multiplying or dividing by 10, 100 or 1000

**Use related facts to divide TU by 0.t**

e.g.  $56 \div 0.8 =$

Using related facts in a division trio could help with this calculation:



Children should be able to explain that because 0.8 is ten times smaller than 8, the answer to  $56 \div 0.8$  will be ten times greater than the answer to  $56 \div 8$  because there will be ten times more groups.

*Examples of calculations*

$21 \div 0.7$

$36 \div 0.9$

$48 \div 0.4$

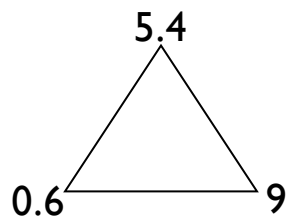
*Prerequisite skills:*

- Recall multiplication tables
- Understand division as repeated subtraction
- Understand the effect of multiplying or dividing by 10

**Use related facts to divide 0.th by 0.t**

e.g.  $0.54 \div 0.6 =$

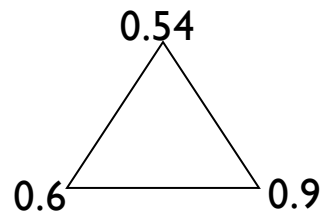
The following division trio from Year 5 could be used as a starting point for this calculation:



Children should be able to explain that 5.4 is ten times smaller than 54 and 0.6 is ten times smaller than 6. This means that both numbers have been scaled down by the same amount, so the relationship

between the numbers stays the same. The answer to  $5.4 \div 0.6$  will therefore be 9 because there are 9 groups of 0.6 in 5.4

The following division trio could then be derived:



Children should be able to explain that because 0.54 is ten times smaller than 5.4, the answer to  $0.54 \div 0.6$  will be ten times smaller than 9. They can then use their understanding of dividing by 10 to calculate this.

*Examples of calculations*

$$0.32 \div 0.4$$

$$0.64 \div 0.8$$

$$0.45 \div 0.9$$

*Prerequisite skills:*

- Recall multiplication tables
- Understand division as repeated subtraction
- Understand the effect of multiplying and dividing by 10

### **Use related facts to divide by 50**

Dividing by 50 is the same as dividing by 100 and then doubling because 50 is half of 100. Children can investigate this using simple calculations.

e.g.  $200 \div 100 = 2$

$$2 \times 2 = 4$$

So  $200 \div 50 = 4$

Children can then use this in more complex calculations.

e.g.  $3200 \div 50 =$

$$3200 \div 100 = 32$$

$$32 \times 2 = 64$$

So  $3200 \div 50 = 64$

*Examples of calculations*

$$4100 \div 50$$

$$7800 \div 50$$

$$530 \div 50$$

*Prerequisite skills:*

- Understand the effect of dividing by 100
- Double numbers including those with one decimal place

### **Use related facts to divide by 25**

Dividing by 25 is the same as dividing by 100 and then multiplying by 4 because 25 is one quarter of 100. Children can investigate this using simple calculations.

e.g.  $200 \div 100 = 2$

$$2 \times 4 = 8$$

So  $200 \div 25 = 8$

Children can then use this in more complex calculations.

e.g.  $4800 \div 25 =$

$$4800 \div 100 = 48$$

$$48 \times 4 = 192$$

So  $4800 \div 25 = 192$

*Examples of calculations*

$$3200 \div 25$$

$$7600 \div 25$$

$$360 \div 25$$

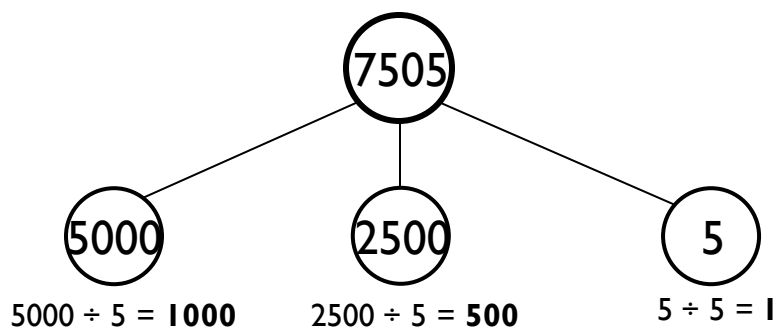
*Prerequisite skills:*

- Understand the effect of dividing by 100
- Multiply numbers up to one decimal place by 4

**Use partitioning to divide ThHTU by a one-digit number**

Building on their understanding of using partitioning to divide TU by a one-digit number from Year 4, children decide how to partition ThHTU to help them divide it by a one-digit number.

e.g.  $7505 \div 5 = 1501$



The diagram above illustrates the way children should be thinking about dividing using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

$5035 \div 5$  By partitioning into 5000 and 35

$1236 \div 4$  By partitioning into 1200 and 36

$9240 \div 6$  By partitioning into 6000 and 3000 and 240

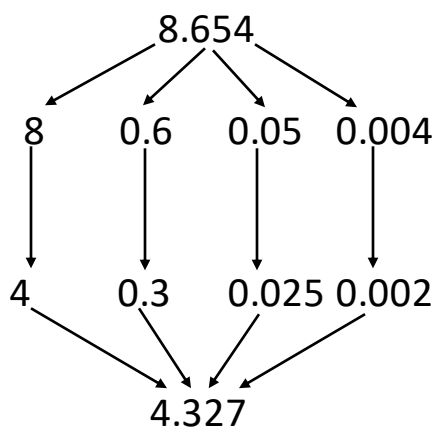
*Prerequisite skills:*

- Recall multiplication tables
- Understand division as repeated subtraction
- Partition four-digit numbers in different ways

**Use partitioning to halve any number including to three decimal places**

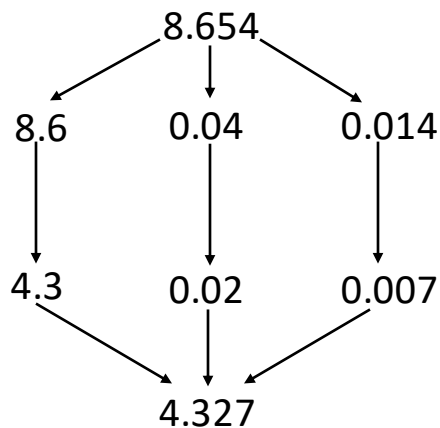
Children should be encouraged to decide the best way to partition a number to halve it.

e.g. Find half of 8.654



An alternative way of partitioning would be:

Find half of 8.654



The diagrams above illustrate the way children should be thinking about halving using partitioning, but it is not necessary for them to record in this way if it is not helpful to the child.

*Examples of calculations*

Find half of 4.684

Find half of 12.826

Find half of 6.942

Find half of 15.674

Find half of 478 612

*Prerequisite skills:*

- Partition numbers (including in different ways for efficiency)
- Use related facts to halve a multiple of a thousandth, hundredth, tenth, ten, hundred and thousand
- Recombine multiples of one, ten, hundred and thousand
- Recombine multiples of a tenth, hundredth and thousandth